Second Practice Exam for Math 403(02) (Spring 2020, Rutgers University, Dr. Z.)

NAME: (print!) ____

Please Email DrZcomlex@gmail.com by May 5, 2020, 1:00pm
[This is the mandatory Homework, but for your own good]
Version of May 3 (please discard previous version)
Email a .txt file attachment or a simple email message with the answers and explanations in Plain English
pracF1FirstNameLastName.txt

Open book (only the class textbook), and open notes (my notes). Any other help (from people or the internet) will be cheating.
EXPLAIN EVERYTHING IN PLAIN ENGLISH
Every question is worth 15 points except the last one that is worth 20 points

1. Find a Laurent series valid in the punctured plane $0 < |z|$ of the function

$$f(z) = \cos z^2 + e^{\frac{1}{z}}$$

2. Describe the locus of the points z satisfying $|z+1+i| = |z-1-i|$.

3. Define $\cos z$ for complex numbers z, in terms of e^z, and prove that

$$\cos 3z = 4\cos^3 z - 3\cos z \quad .$$

4. Recall that if $z = x + iy$ is a complex number, then its **complex conjutagte**, $\bar{z}$ is defined to be $\bar{z} = x - iy$. Find the region in the complex plane where

$$f(z) = \frac{\bar{z}}{(\bar{z}+i)(\bar{z}+2i)} \quad ,$$

is continous. Is it analytic there?

5. Find the circle of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{5^n}{n} (z-1-i)^n$$

6. Compute the line integral (in R^2)

$$\int_\gamma xy \, dx \quad ,$$

if γ is the line segment from $(2,0)$ to $(0,2)$.

7. [Added May 3, 2020: the previous problem was too hard for a final, this problem is more typical] Use Cauchy's formula to compute

$$\int_0^{2\pi} \frac{dt}{2-\cos t} \quad .$$

8. Find the interior, closure, and boundary of the following set in R^2

$$\{(x,y)\,:\, 0 \le x < 1 \quad , \quad 3 < y \le 5\} \quad .$$

9. [Added May 3, 2020: the previous problem was too hard for a final, this problem is more typical]
Find the Taylor polynomial of degree three around $z=0$ of

$$f(z) \,=\, (1+z)^{\frac{1}{3}} \quad .$$

10. Find a Laurent series valid in the annulus $1 < |z| < 2$ of

$$f(z) \,=\, \frac{1}{z^2-3z+2} \quad .$$

11. Find all the roots of the equation

$$z^3 - z^2 - 2iz + 2i \,=\, 0 \quad .$$

12. Use residues to compute the real integral

$$\int_{-\infty}^{\infty} \frac{x^2}{16+x^4}\, dx$$

13. By using the Taylor polynomial of degree 2 around zero of $f(z)=(1+z)^{-\frac{1}{3}}$ approximate

$$\left(\frac{10}{10+i}\right)^{\frac{1}{3}} \quad .$$