First Practice Exam for Math 403(02) (Spring 2020, Rutgers University, Dr. Z.)

NAME: (print!) ---------------------------------------

Please Email DrZcomlex@gmail.com by May 1, 2020, 3:00pm
This is also the Homework assigned on Tue. April 28, 2020, Due May 1, 2020, 3:00pm
Email a .txt file attachment of the format
pracF1FirstNameLastName.txt by May 1, 3:00pm to DrZcomplex@gmail.com

Open book (only the class textbook), and open notes (my notes). Any other help (from people or the internet) will be cheating.
EXPLAIN EVERYTHING IN PLAIN ENGLISH
Every question is worth 15 points except the last one that is worth 20 points

1. Suppose that you have the equation

$$\frac{1}{Z^2} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5}$$

If

$$Z_1 = 2i \quad , \quad Z_2 = 3i \quad , \quad Z_3 = -2i \quad , \quad Z_4 = -3i \quad , \quad Z_5 = i$$

Find Z.

Ans. to 1: The value(s) of Z is (are):

2. Find the contour integral of

$$f(z) = \int_\Gamma \frac{z^9}{(z-1)(z-i)}\, dz$$

If (a) Γ is the contour $|z-1-i| = \frac{1}{5}$

(b) Γ is the contour $|z| = 5$

(c) Γ is the contour $|z-i| = \frac{1}{2}$

(d) Γ is the contour $|z-1| = \frac{1}{2}$

Ans. to 2) a) b) c) d)

3. By using the real form of Green's theorem

$$\int_\Gamma \{udx + vdy\} = \int\int_\Omega (v_x - u_y)\, dx\, dy \quad ,$$

(with the appropriate conditions)

Compute the (real) line integral

$$\int_\Gamma (3x^6 \,+\, 3y)\, dx \,+\, (11x \,+\, 7y^3)\, dy \quad ,$$

where Γ is the contour, transversed **clockwise** that is the boundary of the region in the xy-plane

$$\{\, (x, y) \,:\, 100 \leq x \leq 120 \quad , \quad 1000 \leq y \leq 1003 \,\} \quad .$$

Ans. to 3:

4. By using the Taylor polynomial of degree 3 of $f(z) = \sin z$, find an approximation to $\sin(\frac{i}{2})$.

Ans. to 4:

5. Find the Taylor polynomial of degree 2 around $z_0 = i$ of $f(z) = z^4$

Ans. to 5:

6. [Added May 1, 2020: Rohan Rele noticed that the upper semi-circle is not $\{z : |z| = 5 \quad , \quad Re\, z \geq 0\}$ as previously stated but $\{z : |z| = 5 \quad , \quad Im\, z \geq 0\}$. This is now corrected. If this would hapeend in the Final, you would have gotten credit if you did the right semi-circe]

Find the change of argument of

$$f(z) = \frac{e^z (z - i)(z - 2i)(z - 3i)}{(z - 10i)} \quad ,$$

(a) as z goes along, counterclockwise, the upper semi-disc

$$\{z : |z| = 5 \quad , \quad Im\, z \geq 0\} \cup \{x + 0 \cdot i \,:\, -5 \leq x \leq 5\}$$

(b) as z goes along, **clockwise**, the upper semi-disc

$$\{z : |z| = 20 \quad , \quad Im\, z \geq 0\} \cup \{x + 0 \cdot i \,:\, -20 \leq x \leq 20\}$$

Ans. to 6(a): **Ans. to 6(b)**:

7. which of the following functions $f(z) = f(x + iy)$ are analytic. Explain

$$(a) \quad f(x+iy) = 2x - 3iy \quad , \quad (b) \quad f(x+iy) = -ix + y \quad ,$$
$$(c) \quad f(x+iy) = x^2 - y^2 + i(2xy) \quad , \quad (d) \quad f(x+iy) = x^2 + y^2 + i(2xy) \quad ,$$

Ans. to 7: The following ones are: (put the subset of $\{a, b, c, d\}$):

8. Use Rouché's theorem to prove that all the zeros of the function $f(z) = z^5 - i$ lie in the annulus $0.99 < |z| < 1.01$. Explain!

Sol. to 8:

9 Which of the following statements are **obviously wrong**, without actually doing the problem. EXPLAIN!

a) The change of argument, of the function $f(z) = e^{z^3}$ as it transverses, counter-clockwise, the circle $|z| = 10$ is 2π.

b) The change of argument, of the function $f(z) = \cos z$ as it transverses, counter-clockwise, the circle $|z| = 100$ is 3π.

c) The change of argument, of the function $f(z) = \frac{1}{z^9+1}$ as it transverses, **clockwise**, the circle $|z| = 5$ is 18π.

d) The change of argument, of the function $f(z) = \tan z$ as it transverses, counter-clockwise, the circle $|z| = 1$ is 2π.

Ans. to 9: The following ones are **obviosuly wrong**:

10. A certain entire function maps the region $\{z : |z| < 1\}$ **onto** the region $\{z : |z - 3| \leq 2\}$. What can you say about such a function. EXPLAIN!

Ans. to 10:

11. Evaluate the following integral

$$\int_0^{2\pi} (2i + 10\, e^{it})^9 \, dt$$

Ans. to 11:

12: You are told that a certain function, analytic in $|z| < 10$ is such that $f(0) = 0$, $|f(z)| \leq 100$ and $f(5i) = (30 + 40i)$. Find the exact expression for $f(z)$.

Ans. to 12:

13: Find the exact value of the contour integral

$$\int_{|z-2i|=5} \frac{e^{z^2}}{(z-i)^2}\, dz$$

Ans. to 13: