Solutions to Dr. Z.'s Intro to Complex Variable Attendance Quiz for the Review Session

1. Which of the following statements are **obviously wrong**, without actually doing the problem. EXPLAIN!

1. $\int_0^{2\pi} \frac{1}{1+\sin^{10} t}\, dt \;=\; -\frac{\pi}{10}$

Sol. to 1: **NONSENSE**, the integrand is always positive, so the integral can't be negative.

2. $\int_{|z|=2} \frac{z^3}{(z^2-4)^3(z+1)}\, dz \;=\; 10\pi\, i$

Sol. to 2: **NONSENSE**, the integrand bas a pole (i.e. **blows up**) on the contour of integration, namely at z=2 and z=-2, so the integral is **UNDEFINED**.

3. The change of argument, of the function $f(z) = z^5 - z^3 + 1 + e^z$ as it transverses, counter-clockwise, the contour that goes from 0 to 10, 10 to $10+30i$, $10+30i$ to $30i$, and $30i$ back to 0 is 9π.

Sol. to 3: **NONSENSE**. By the **argument principle**, the change of argument of a function $f(z)$ as you travel counter-clockwise around a countor **divided by** 2π, equals

"the number of zeros minus the number of poles of that function inside the contour."

This number is an integer! (and if the function happens to be analytic, a non-negative integer). 9π divided by 2π is $\frac{9}{2}$, that is **not** an integer. So this can never happen.

4. The change of argument, of the function $f(z) = z^5 - z^3 + 1$ as it transverses, counter-clockwise, the contour $|z| = 10^{1000000}$ is 8π.

Sol. to 4: **NONSENSE**, but for a more subtle reason. $8\pi/(2\pi) = 4$ is an integer, and even a positive integer, **but** since $10^{1000000}$ is so big, the disc $|z| \leq 10^{1000000}$ includes all the zeros of $f(z)$, and since $f(z)$ is a polynomial of degree **five**, by the **fundamental theorem of algebra**, there are exactly five zeros (counting multiplicity). Hence this is nonsense, for a less obvious reason.

5. The change of argument of $f(z) = e^z - z^3$ as z transverses, counter-clockwise, the contour $|z| = 5$ is -4π

Sol. to 5: **NONSENSE**. $f(z)$ is analytic, so it has no poles. Hence the number of zeros minus the number of poles is a non-negative integer, so it can't be $\frac{-4\pi}{2\pi} = -2$.

6. The change of argument of $f(z) = \frac{e^z}{(z-1)(z-2)}$ as z transverses, counter-clockwise, the contour $|z| = 3$ is 10π.

Sol. to 6: **NONSENSE**. Since the top of $f(z)$, e^z famously has no zeros, neither does $f(z)$, so $f(z)$ only has poles (in fact two poles), so the change of argument divided by 2π is a negative integer (in fact -2), so it can't be 5.

7. In order to approximate $(1+\frac{3}{2}\,i)^{\frac{1}{3}}$ you use the Taylor expansion of $(1+z)^{\frac{1}{7}}$ up to the tenth-power and then plug-in $z=\frac{3}{2}\,i$.

Sol. to 7: **NONSENSE**. The **radius of convergence** of the Taylor expansion of $(1+z)^{\frac{1}{7}}$ around $z=0$ is 1, and $|z|=\frac{3}{2}>1$, so if you try to plug-it in you get **garbage**.