Solutions to Dr. Z.'s Intro to Complex Variable Attendance Quiz for Lecture 4

1. Find all the points of continuity of the given function

$$f(z) = \begin{cases} \frac{z^5-1}{z-1}, & z \neq 1 \\ 3, & z = 1 \end{cases} \quad .$$

Sol. 1: Since $\frac{z^5-1}{z-1}$ is always continuous except at $z = 1$ (where is is undefined), $f(z)$ is definitely continuous when $z \neq 1$. The limit of $f(z)$ as z goes to $z = 1$ is, by L'Hôpital's rule $5z^4/1|_{z=1} = 5$. Since

$$\lim_{z \to 1} f(z) = 5 \quad ,$$

while $f(1) = 3$, it folllows that $f(z)$ is **not** continuous at $z = 1$.

Ans. to 1: The set of points where $f(z)$ is continuous is

$$\{z \in C \, : \, z \neq 1\} \quad .$$

2. Determine whether the given series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1}{3+i^n} \quad .$$

Sol. of 2: The terms of the **sequence** $\frac{1}{3+i^n}$ are $\frac{1}{4}, \frac{1}{3+i}, \frac{1}{2}, \frac{1}{3-i}$, and back to $\frac{1}{4}$ etc. So the sequence is divergent, and definitely does not go to 0, hence by the **divergence test** that says that a **necessary** condition for the **series** to converge is that the corresponding **sequence** converges to 0 it follows that the series **diverges**.

Ans. to 2: The series diverges because of the divergence test.