Solutions to Dr. Z.'s Intro to Complex Variable Attendance Quiz for Lecture 3

1. For the set $\{z = x + iy : x > 2 \quad , \quad y < -3\}$,

(a) Describe the interior and the boundary (b) State whether the set is open, closed, or neither (c) State whether the interior of the set is connected

Sol. to 1: The set is a quarter plane strictly to the right of the vertical line $x = 2$ and strictly below the horizontal line $y = -3$. (a) Because of the **strict** inequalities the **interior** is the set itself. Its **boundary** consists of a union of two half-lines

$$\{(2, y) : \leq -3\} \cup \{(x, -3) : x \geq 2\}$$

(b) Since the interior of the set equals itself, it is an **open** set.

(c) The interior of the set is connected, since you cam go from any of its points to any other point, without leaving the set.

2. Let

$$\Omega_1 = \{z : 4 < |z| < 8 \quad and \quad Re\, z > -2\} \quad , \quad \Omega_2 = \{z : 4 < |z| < 8 \quad and \quad Re\, z < 2\} \quad .$$

Show that Ω_1 and Ω_2 are domains, but $\Omega_1 \cap \Omega_2$ is not. Draw all these sets.

Sol. to 2: Ω_1 is the part of the annulus $4 < |z| < 8$ to the right of the line $x = -2$, and is both open and connected.

Ω_2 is the part of th the annulus $4 < |z| < 8$ to the left of the line $x = 2$, and is both open and connected.

But $\Omega_1 \cap \Omega_2$ is the part of the annulus between $x = -2$ and $x = 2$ and consists of two disconnected components, a top one and a bottom one. So even though it is **open** (being the intersection of two open sets) it is **not** connected, hence it is **not** a domain, since a domain, by definition has to be both open and connected.