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1. $\int_{-\infty}^{\infty} \frac{x^2}{x^4+16}\,dx \longrightarrow \int_{\gamma R} \frac{z^2}{z^4+16}\,dz$

$z^4+16 = (z^2+4i)(z^2-4i) = (z+a)(z-a)(z+b)(z-b)$

$a = \sqrt{4i} = 2e^{i\frac{\pi}{4}} = \sqrt{2} + i\sqrt{2}$

$b = \sqrt{-4i} = i\sqrt{4i} = -\sqrt{2} + i\sqrt{2}$

only $a = \sqrt{2} + i\sqrt{2}$ & $b = -\sqrt{2} + i\sqrt{2}$ lie in the upper plane

$\operatorname{Res}(f(z);a) = \frac{a^2}{2a(a^2-b^2)} \leftarrow \left(\frac{z^2}{(z+a)(z^2-b^2)}\right)$

$= \frac{4i}{2a(4i-(-4i))} = \frac{4\cancel{i}}{2\cdot 8\cancel{i}}\left(\frac{1}{\sqrt{2}+i\sqrt{2}}\right) = \frac{1}{4}\left(\frac{1}{4}\right)(\sqrt{2}-i\sqrt{2}$

$\operatorname{Res}(f(z);b) = \frac{b^2}{2b(b^2-a^2)} = \frac{-4i}{2b(-4i-4i)} = \frac{-4\cancel{i}}{2\cdot(-8\cancel{i})}\frac{1}{(-\sqrt{2}+i\sqrt{2})}$

$= \frac{1}{4}\left(\frac{1}{4}\right)(-\sqrt{2} - i\sqrt{2})$

$\int_{-\infty}^{\infty} \frac{x^2}{x^4+16}\,dx = 2\pi i\left(\frac{1}{16}(\cancel{\sqrt{2}} - i\sqrt{2}) + \frac{1}{16}(\cancel{-\sqrt{2}} - i\sqrt{2})\right)$

$2\pi\cancel{i}\left(\frac{2}{16}(-\cancel{i}\sqrt{2})\right) = \frac{4\sqrt{2}}{16}\pi = \boxed{\frac{\sqrt{2}}{4}\pi}$