Midterm Exam for Math 403(02) (Spring 2020, Rutgers University, Dr. Z.), April 16, 2020, 1:40-3:00pm DST

NAME: (print!) ________________________________

Please Email DrZcomlex@gmail.com by April 16, 2020, 3:05pm
and
Subject: mt
Call the file (RESPECT capitilization)
mtFirstNameLastName.pdf (preferred) OR mtFirstNameLastName.jpg (ONE file) OR
mtFirstNameLastNamePageNumber.jpg , Number=1,2,3,4 OR
A simple Email message, writing the answers in Plain English and using computereze for math symbols

Open book (only the class textbook), and open notes (my notes). Any other help (from people or the internet) will be cheating.

1. (20 points altogether) In an AC (*alternating current*) electrical circuit if three elements are in **parallel**, and their **impedences** are Z_1, Z_2, Z_3, then their **joint impedence**, let's call it Z, is given by the formula $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$, on the other hand if they are connected in **series**, then $Z = Z_1 + Z_2 + Z_3$. If three circuits have impedences $1+i$, $1-i$, and i, what is the impedence if they are joined

(a) (5 points) in series (b) (15 points) in parallel.

Ans. (a) (b)

2. (20 points altogether) Consider the function

$$f(z) = \frac{z+1}{z^3 - 3z^2 + 2z}$$

(a) (1 point) Find all the zeros of $f(z)$

Ans. to (a): $z =$

(b) (4 points) Find all the poles, and for each of them find the residue

Ans. to (b): $z =$, $Residue =$; $z =$, $Residue =$;
$z =$, $Residue =$.

Compute the countour integral $\int_\gamma f(z)\,dz$ for the following contours
(c) (5 points) $|z| = \frac{1}{2}$ (d)(5 points) $|z| = \frac{3}{2}$ (e)(5 points) $|z| = 10$

Ans. to (c): **Ans. to (d)**: **Ans. to (e)**:

3. (10 points) The Sweatshirt of the Rutgers Mathematics Graduate Student club has the following contour integral

$$\int_{|z-5|=1} \frac{37z(z+1)}{(z-2)(z-5)}\,dz \quad .$$

Find its exact value. Explain everything.

Ans.:

4. (15 points. altogether) (a) (10 points) Find the first three terms (i.e. the terms involving z^n for $n = 0, 1, 2$) of the function

$$f(z) = (1+z)^{\frac{1}{7}} \quad .$$

Ans. to (a):

(b) (3 points) Use this to approximate $(1 + \frac{1}{2}i)^{\frac{1}{7}}$.

Ans. to (b):

(c) (2 points) Can you use part (a) to find an approximation of $(1+2i)^{\frac{1}{7}}$?. Explain!

5. (20 points altogether) For the following functions decide whether there are analytic in the complex plane. Explain!

(a) (10 points)

$$f(x+iy) \;=\; e^x(x\cos y - y\sin y) + ie^x(y\cos y + x\sin y) \quad .$$

(b) (10 points)

$$f(x+iy) = e^x(x + x\cos y - y\sin y) + ie^x(y\cos y + x\sin y) \quad .$$

6. (15 points) Verify the **real form** of Green's theorem (that says that $\int_\Gamma \{udx + vdy\} = \int\int_\Omega (v_x - u_y)\,dxdy$) (with the appropriate conditions) for the following case

$$u(x,y) = x + 2y \quad , \quad v(x,y) = 3x + y \quad ,$$

and the contour Γ is the triangle whose vertices are $(0,0)$, $(1,0)$, $(0,1)$.

7. (10 points) Describe the set of entire functions $f(z)$ such for every complex number z, $|f(z)| \leq 1000000$. Explain!

8. (10 points) Use Rouché's theorem to prove that all the zeros of the function $f(z) = z^5 - z - 3$ lie in the annulus $\frac{11}{10} < |z| < \frac{7}{5}$. Explain!

9 (15 points altogether, 5 points each) Which of the following statements are **obviously wrong**, without actually doing the problem. EXPLAIN!

a) The change of argument, of the function $f(z) = e^{-z}$ a it transverses, counter-clockwise, the circle $|z| = 100$ is 7π.

b) The change of argument, of the function $f(z) = e^{-3z}$ a it transverses, counter-clockwise, the circle $|z| = 100$ is 4π.

c) The change of argument, of the function $f(z) = \frac{1}{(z-1)(z-2)}$ a it transverses, counter-clockwise, the circle $|z| = 5$ is 4π.

10. (10 points) Find the Taylor series about $z = i$ of $f(z) = z^2$.

11. (5 points) Compute. Explain.

$$\frac{1}{2\pi i} \int_{|z|=10} \frac{\sin\, z}{(z - \frac{\pi}{6})^6}\, dz$$