

Dr. Z.'s Intro to Complex Variables Homework assignment 22

Version of April 24, 2020 (Correcting a typo, thanks to Vishal Patel)

Due: April 30, 2020, 1:00pm. Please email (either scanned handwriting, or .txt, or .pdf) to

ctk47@math.rutgers.edu

with an attachment called

hw22FirstNameLastName.pdf [or hw22FirstNameLastName.jpg]

and Subject: hw22

1. State and prove Schwarz's lemma.
2. State and prove the Mean Value Theorem.
3. Explain why it is impossible to have an analytic function $f(z)$ such that $f(0) = 0$, $|f(z)| \leq 1000$ in the disc $|z| < 6$ and $|f(3i)| = 550$.
4. Compute the following integrals

$$\text{(a)} \int_0^{2\pi} \cos\left(\frac{\pi}{2} + 5e^{it}\right) dt \quad , \quad \text{(b)} \int_0^{2\pi} e^{i+30e^{it}} dt \quad , \quad \text{(c)} \int_0^{2\pi} \tan\left(\frac{\pi}{4} + \frac{e^{it}}{20}\right) dt \quad .$$

5.:For the following function

$$u(x, y) = x^4 - 6x^2y^2 + y^4 \quad ,$$

- (a) verify that it is harmonic
- (b) Find the absolute maximum values and absolute minimum values, and their locations, of $u(x, y)$ in the region $\{(x, y) : 0 \leq x, y \leq 1\}$.