Dr. Z.'s Intro to Complex Variables Homework assignment 10

1. State and prove the binomial theorem.

2. Let $a_0, a_1, a_2 \ldots$ be complex numbers with

$$\lim_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} = L \quad .$$

Show that the series

$$\sum_{n=0}^{\infty} \frac{a_n}{(z - z_0)^n} \quad ,$$

converges absolutely for all z with $|z - z_0| > L$ and diverges for all z with $|z - z_0| < L$.

3. Find the first 5 terms of the Taylor expansions of the following analytic functions around $z = 0$. In other words find it up to the power z^4

a. $f(z) = e^z \cos z$

b. $f(z) = \sqrt{1 - z}\, e^z$

c. $f(z) = \cos z^5$

d. $f(z) = z \sin z^3$

e. $f(z) = \log(1 + z) \cosh z$