

## Solutions to Real Quiz 9 of Dr. Z.'s Dynamical Models in Biology class

**Name:** Dr. Z.

1. Find all the equilibrium points and stable equilibrium points of the following one-dimensional dynamical system

$$\frac{dx}{dt} = -(x+2)(x+1)(x-1)(x-2)(x-5) .$$

**Sol. to 1:**

The equilibrium points are the solutions of

$$-(x+2)(x+1)(x-1)(x-2)(x-5) = 0 ,$$

where are  $\{-2, -1, 1, 2, 5\}$ .

The derivative of the function on the right is (by the product rule of calculus)

$$-((x+2)'(x+1)(x-1)(x-2)(x-5)+(x+2)(x+1)'(x-1)(x-2)(x-5)+(x+2)(x+1)(x-1)'(x-2)(x-5)+(x+2)(x+1)(x-1)(x-2)'(x-5)+(x+2)(x+1)(x-1)(x-2)(x-5)' =$$

$$f'(x) = -((x+1)(x-1)(x-2)(x-5)+(x+2)(x-1)(x-2)(x-5)+(x+2)(x+1)(x-2)(x-5)+(x+2)(x+1)(x-1)(x-2)(x-5))$$

When  $x = -2$  we have

$$f'(-2) = -(-2+1)(-2-1)(-2-2)(-2-5) = -(-1)(-3)(-4)(-7) > 0 ,$$

since this is positive  $x = -2$  is **stable**.

When  $x = -1$  we have

$$f'(-1) = -((-1+2)(-1-1)(-1-2)(-1-5)) = (-1)(1)(-2)(-6) < 0$$

since this is positive  $x = -1$  is **unstable**.

Similarly, When  $x = 1$  it is stable, when  $x = 2$  it is unstable when  $x = 5$  it is stable.

**Ans:** The stable equilibrium points are  $\{-2, 1, 5\}$ .