

Solutions to Real Quiz 9 of Dr. Z.'s Dynamical Models in Biology class

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1. Find all the equilibrium points and stable equilibrium points of the following one-dimensional dynamical system

$$\frac{dx}{dt} = -(x+2)(x+1)(x-1)(x-2)(x-5) \quad .$$

Sol. to 1:

The equilibrium points are the solutions of

$$-(x+2)(x+1)(x-1)(x-2)(x-5) = 0 \quad ,$$

where are $\{-2, -1, 1, 2, 5\}$.

The derivative of the function on the right is (by the product rule of calculus)

$$\begin{aligned} & -((x+2)'(x+1)(x-1)(x-2)(x-5) + (x+2)(x+1)'(x-1)(x-2)(x-5) + (x+2)(x+1)(x-1)'(x-2)(x-5) + (x+2)(x+1)(x-1)(x-2)'(x-5) + (x+2)(x+1)(x-1)(x-2)(x-5)') \\ f'(x) = & -((x+1)(x-1)(x-2)(x-5) + (x+2)(x-1)(x-2)(x-5) + (x+2)(x+1)(x-2)(x-5) + (x+2)(x+1)(x-1)(x-2)(x-5)') \end{aligned}$$

When $x = -2$ we have

$$f'(-2) = -(-2+1)(-2-1)(-2-2)(-2-5) = -(-1)(-3)(-4)(-7) > 0 \quad ,$$

since this is positive $x = -2$ is **stable**.

When $x = -1$ we have

$$f'(-1) = -((-1+2)(-1-1)(-1-2)(-1-5)) = (-1)(1)(-2)(-6) < 0$$

since this is positive $x = -1$ is **unstable**.

Similarly, When $x = 1$ it is stable, when $x = 2$ it is unstable when $x = 5$ it is stable.

Ans: The stable equilibrium points are $\{-2, 1, 5\}$.