Solutions to Real Quiz 7 of Dr. Z.'s Dynamical Models in Biology class

Name: Dr. Z.

1. (a): (3 points) . Prove that $(0,0)$ is a steady-state of the first-order vector recurrence

$$a_1(n+1) = \frac{a_1(n)}{2+a_2(n)} \quad ,$$

$$a_2(n+1) = \frac{a_2(n)}{3+a_1(n)} \quad .$$

Explain!

Sol. to 1:

The **unerlying** transformation from R^2 to R^2 is

$$(x,y) \to \left(\frac{x}{2+y}, \frac{y}{3+x}\right) \quad .$$

Sol of 1(a):

Plugging in $x=0$ and $y=0$ on the right side gives $(0,0)$, so $(0,0)$ is a fixed point of the transformation, and so $a_1(n)=0, a_2(n)=0$ is a **steady-state**.

Sol of 1(b): The **jacobian matrix is**

$$J(x,y) = \begin{bmatrix} \frac{1}{2+y} & -\frac{x}{(2+y)^2} \\ -\frac{y}{(3+x)^2} & \frac{1}{3+x} \end{bmatrix} \quad .$$

Plugging-in $x=0$, $y=0$ we have

$$J(0,0) = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \quad .$$

The **characteristic equation** is $(\lambda - \frac{1}{2})(\lambda - \frac{1}{3}) = 0$, whose solutions are $\frac{1}{2}$ and $\frac{1}{3}$. So the **eigenvalues** are $\frac{1}{2}$ and $\frac{1}{3}$. Both of them have **asbsolute value** less than 1, so this proves that $(0,0)$ is a stable steady-state.

Ans. to 1(b): $(0,0)$ is a stable steady-state.