Solutions to Real Quiz 6 of Dr. Z.'s Dynamical Models in Biology class

1. (5 points) Find the steady-states of the second-order recurrence

$$a(n+2) = \frac{1+3a(n+1)+4a(n)}{2+a(n+1)+5a(n)} \quad .$$

Sol. to 1: We replace $a(n), a(n+1), a(n+2)$ by z getting the equation

$$z = \frac{1+3z+4z}{2+z+5z} \quad .$$

Simplifying

$$z = \frac{1+7z}{2+6z} \quad .$$

Moving everying to the left

$$z - \frac{1+7z}{2+6z} = 0 \quad .$$

Multiplying by $2+6z$

$$z(6z+2) - (1+7z) = 0 \quad .$$

Expanding

$$6z^2 - 5z - 1 = 0 \quad .$$

Factoring

$$(6z+1)(z-1) = 0 \quad ,$$

so there are two roots $z = -\frac{1}{6}$ and $z = 1$.

Ans. to 1: The steady-states of this recurrence are $z = -\frac{1}{6}$ and $z = 1$.

2. (5 points) Convert the fifth-order recurrence

$$a(n+5) = \frac{1+4a(n+4)+6a(n)}{2+4a(n+3)+5a(n+1)} \quad .$$

to a first-order vector recurrence for the vector:

$$\mathbf{x}(n) = \begin{bmatrix} a(n+4) \\ a(n+3) \\ a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix} \quad .$$

Sol to 2: Using the reciple we have

$$\mathbf{x}(n+1) = \mathbf{F}(\mathbf{x}(n)) \quad ,$$

where $\mathbf{F}$ is

$$(x_1, x_2, x_3, x_4, x_5) \to \left(\frac{1+4x_1+6x_5}{2+4x_2+5x_4}, x_1, x_2, x_3, x_4 \right) \quad .$$