Solutions to Real Quiz 5 of Dr. Z.'s Dynamical Models in Biology class

Name: (print!) Dr. Z.

1.: (4 points) Let $a(n)$ be the discrete function defined on $0 \leq n \leq 99$ satisfying:

Apology: The previous version solved a problem with $a(99) = 3$. Here is the correct version.

$$a(n+2) - a(n) \ = \ 0 \quad ; \quad a(0) = 1 \quad , \quad a(99) = 2 \quad .$$

What is $a(40)$? What is $a(41)$?

Sol. to 1: The characteristic equation is:

$$z^2 - 1 = 0 \quad .$$

Factoring

$$(z-1)(z+1) = 0 \quad .$$

Hence the roots are $z_1 = 1$, and $z_1 = -1$. Hence the general solution is

$$a(n) = c_1 \cdot 1^n + c_2 \cdot (-1)^n = c_1 + c_2 \cdot (-1)^n \quad .$$

Using $a(0_=1$ and $a(99) = 2$ we have the two equations

$$\{c_1 + c_2 = 1 \quad , \quad c_1 - c_2 = 2\} \quad .$$

Solving, we have:

$$c_1 = \frac{3}{2} \quad , \quad c_2 = -\frac{1}{2} \quad .$$

Pluging in the general solution, we have:

$$a(n) = \frac{3}{2} - \frac{1}{2} \cdot (-1)^n$$

Finally we have $a(40) = 1$, $a(41) = 2$.

2. (4 points). In a gambler's ruin game you win a dollar the maximal capital is 10 dollars. You currently have 5 dollars.

(i): (1 point) How likely are you to exit a winner if the prob. of winning a dollar in any one round is 0.5 (and hence the prob. of losing a dollar is 0.5)?

Sol. to 1(i): The formuls for the fair case is $\frac{n}{L}$, so the the prob. of exiting a winner is $\frac{5}{10} = \frac{1}{2}$.

(ii): (3 point) How likely are you to exit a winner if the prob. of winning a dollar in any one round is 0.4 (and hence the prob. of losing a dollar is 0.6)?

Sol. to 2(ii): The formuls for the the loaded case is

$$\frac{1-(q/p)^n}{1-(q/p)^L} \quad ,$$

where $q = 1 - p$. Here $p = 0.4$ so $q = 0.6$ and $q/p = 3/2$ amd the prob. of exiting a winner is

$$\frac{1-(3/2)^5}{1-(3/2)^{10}} = \frac{32}{275} = 0.116363636\ldots \quad ,$$

Ans. to 2(ii): The prob. of exiting a winner is 0.116363636.

3. (2 points) What is the expected duration of a fair Gambler's Ruin game if the maximal capital is 200 dollars and you currently have 100 dollars?

Sol. of 3: The formula for the expected duration is $n(L-n)$ so $100\cdot(200-100) = 100\cdot 100 = 10000$.

Ans. of 3: The expected duration is 10000 rounds.