

Solutions to Real Quiz 3 of Dr. Z.'s Dynamical Models in Biology class

1. (5 points altogether)

a. (2 points) Convert the recurrence

$$6a(n-1) + a(n+1) + 5a(n) = 0 \quad ,$$

into **standard form** where $a(n+2)$ is expressed in terms of $a(n+1), a(n)$.

Sol. of 1(a):: Replacing n by $n+1$ we have

$$6a(n) + a(n+2) + 5a(n+1) = 0 \quad ,$$

Moving $a(n+2)$ to the left we get

$$a(n+2) = -5a(n+1) - 6a(n)$$

b. (3 points)

Abbreviating

$$\mathbf{a}(n) = \begin{bmatrix} a(n+1) \\ a(n) \end{bmatrix} \quad .$$

Find the 2×2 matrix, let's call it A such that

$$\mathbf{a}(n+1) = A\mathbf{a}(n) \quad .$$

Sol. to 1(b)

$$A = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix}$$

2. (5 points) In a certain species only one-year-olds, two-year-olds are fertile. We have

- zero-year-olds can't have babies
- Every 1-year-old female makes 1.3 babies on average
- Every 2-year-old female makes 0.9 babies on average

We also know

- The probability that a zero-year-old will survive the year is 0.7
- The probability that a one-year-old will survive the year is 0.8

Set up the **Leslie matrix**

Sol.to 2 The **fertility vector** is $[0, 1.2, 0.9]$. The **survival vector** is $[0.7, 0.8]$ the Leslie matrix has all zeroes except the first row that is the fertility vector and the sub-diagonal that is the survival vector.

$$\begin{bmatrix} 0 & 1.2 & 0.9 \\ 0.7 & 0 & 0 \\ 0 & 0.8 & 0 \end{bmatrix}$$