

Solutions to Real Quiz 2 of Dr. Z.'s Dynamical Models in Biology class

Name: Dr. Z.

1. (7 points altogether)

a. (5 points) Find all the eigenvalues of the matrix

$$\begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix}$$

Sol. to 1(a):

The characteristic matrix is

$$\begin{bmatrix} 3 - \lambda & 6 \\ 6 & 3 - \lambda \end{bmatrix}$$

Taking determinants, the **characteristic equation** is

$$(3 - \lambda)^2 - 6^2 = 9 - 6\lambda + \lambda^2 - 36 = 0$$

Simplifyig:

$$\lambda^2 - 6\lambda - 27 = 0$$

Factorizing

$$(\lambda - 9)(\lambda + 3) = 0 \quad .$$

Hence

Ans. to 1a: The eigenvalues are -3 and 9 .

b. (2 points) Find the corresponding eigenvector of the larger eigenvalue.

Sol. to 1b: We only need to do the **larger** eigenvalue, $\lambda = 9$ (shame on you, those who did also the other one). let $\begin{bmatrix} a \\ b \end{bmatrix}$ be a tentative eigenvector corresponind to $\lambda = 9$. Then

$$\begin{bmatrix} 3 - 9 & 6 \\ 6 & 3 - 9 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad .$$

So

$$\begin{bmatrix} -6 & 6 \\ 6 & -6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad .$$

We have to solve the system

$$\{-6a + 6b = 0, 6a - 6b = 0\} \quad .$$

As expected the second equation is equivalent to the first, so $b = a$ and the general eigenvector if

$$\begin{bmatrix} a \\ a \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad .$$

Taking $a = 1$ we have:

Ans. to 1b: An eigenvector corresponding to the eigenvalue 9 is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2. (3 points) Solve the following recurrence with the given initial conditions

$$a(n) - a(n-2) = 0 \quad ; \quad a(0) = 2 \quad , \quad a(1) = 0 \quad .$$

Sol. to 2':

The *characteristic equation* is

$$z^2 - 1 = 0 \quad .$$

Factoring

$$(z-1)(z+1) = 0 \quad .$$

Hence the **general solution** is

$$a(n) = c_1 \cdot (1)^n + c_2 \cdot (-1)^n = c_1 + c_2 \cdot (-1)^n \quad .$$

Using the **initial conditions**

$$a(0) = 2 = c_1 + c_2 \quad , \quad a(1) = 0 = c_1 - c_2 \quad ,$$

whose solution is $c_1 = 1$, $c_2 = 1$. Going back to the general solution we have

Ans. to 2:

$$a(n) = 1 + (-1)^n \quad .$$