

Solutions to Real Quiz 1 of Dr. Z.'s Dynamical Models in Biology class

1. (3 points) Compute $x(4)$ of the sequence satisfying the recurrence equation

$$x(n) = 4x(n-1) - 5x(n-2) + x(n-3) \quad , n \geq 3$$

subject to the initial conditions

$$x(0) = 1 \quad , \quad x(1) = 1 \quad , \quad x(2) = 1 \quad .$$

Sol. to 1: First we must compute $x(3)$, we have, by plugging-in $n = 3$ into the recurrence

$$x(3) = 4x(2) - 5x(1) + x(0) = 4 \cdot 1 - 5 \cdot 1 + 1 = 0$$

Now we plug-in $n = 4$:

$$x(4) = 4x(3) - 5x(2) + x(1) = 4x(3) - 5x(2) + x(1) = 4 \cdot 0 - 5 \cdot 1 + 1 = -4 \quad .$$

Ans. to 1: $x(4) = -4$

2. (7 points) Solve explicitly the differential equation

$$y''(x) - 4y(x) = 0 \quad .$$

with initial conditions

$$y(0) = 2 \quad , \quad y'(0) = 0 \quad .$$

Sol. to 2: The **characteristic equation** is

$$z^2 - 4 = 0 \quad .$$

Factoring

$$(z - 2)(z + 2) = 0 \quad .$$

So there are two **distinct** roots: $z_1 = -2$ and $z_2 = 2$.

So the **general solution** is

$$y(x) = c_1 e^{-2x} + c_2 e^{2x} \quad .$$

For future reference

$$y'(x) = -2c_1e^{-2x} + 2c_2e^{2x} \quad .$$

Now we take advantage of the initial conditions

$$2 = y(0) = c_1e^{-2 \cdot 0} + c_2e^{2 \cdot 0} = c_1 + c_2$$

$$0 = y'(0) = -2c_1e^{-2 \cdot 0} + 2c_2e^{2 \cdot 0} = -2c_1 + 2c_2 \quad .$$

We have a system of two linear equations for the unknowns c_1, c_2 .

$$c_1 + c_2 = 2 \quad , \quad , -2c_1 + 2c_2 = 0 \quad ,$$

whose solution is $c_1 = 1, c_2 = 1$. Going back to the general solution we have

Ans. to 2:

$$y(x) = e^{2x} + e^{-2x}$$

(or $y(x) = 2 \cosh 2x$, that's how Maple would give the answer).