Solutions to Attendance Quiz for Lecture 9 of Dr. Z.'s Dynamical Models in Biology class

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1.: A gambler hits a **fair** casino, where the maximuam gain is 100 dollars, with 30 dollars, and must leave as soon as he either made 100 dollars or got broke.

(i) How likely is he to exit with 100 dollars?

Sol. to 1(i): By the general formula n/L with $n = 30$ and $L = 100$ we have

Ans. to 1(i): The probability if exiting the casino a winner is $30/100 = 0.3$

(ii) How many rounds should he expect to play before he has to leave the casino?

Sol. to 1(ii): By the general formula $n(L - n)$ with $n = 30$ and $L = 100$ we have

Ans. to 1(ii): The expected number of roungds until exiting the casion (either as a winner or loser) is $30 \cdot (100 - 30) = 2100$.

2. Solve the bounary-value linear-recurrence equation problem

$$a(n+2) \; = \; a(n+1) \; - \; \frac{3}{16}\, a(n) \quad ,$$

valid in $0 \leq n \leq L-2$, subject to the bounardy conditions:

$$a(0) = 0 \quad , \quad a(L) = 1 \quad .$$

Sol. of 2: The characteristic equation is

$$z^2 - z + \frac{3}{16} = 0 \quad .$$

Factoring

$$(z - \frac{1}{4})(z - \frac{3}{4}) = 0 \quad ,$$

so the two roots are $z_1 = \frac{1}{4}$ $z_2 = \frac{3}{4}$.

SO the genearal solution is

$$a(n) = A_1 (\frac{1}{4})^n \; + \; A_2 (\frac{3}{4})^n \quad ,$$

where A_1, A_2 are to be determined.

Taking advantage of the boundary conditions

$$a(0) = 0 = A_1 + A_2 \quad ,$$

$$a(L) = 1 = A_1(\frac{1}{4})^L \; + \; A_2(\frac{3}{4})^L \quad ,$$

From the first equation, $A_2 = -A_1$ and putting it in the second equation we get

$$A_1 \left((\frac{1}{4})^L \; - \; (\frac{3}{4})^L \right) \quad .$$

Hence

$$A_1 = \frac{1}{\frac{1}{4})^L \; - \; (\frac{3}{4})^L} \quad , A_2 = -\frac{1}{\frac{1}{4})^L \; - \; (\frac{3}{4})^L} \quad ,$$

Going back to the general solution, we have

Ans. to 2:

$$a(n) = \frac{(\frac{1}{4})^n - (\frac{3}{4})^n}{(\frac{1}{4})^L - (\frac{3}{4})^L} \quad .$$