Solutions to Attendance Quiz for Lecture 6 of Dr. Z.'s Dynamical Models in Biology class

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1. Suppose that at year $t = 0$ there were 1 0-year-olds, 2 1-year-olds, 3 2-year-olds, 4 3-year-olds.

For the following Leslie matrices answer

(i) How many 0-year-olds at time $t = 3$? (give the Maple command that does it)

(ii) in the long run will the population increase exponentially? decrease exponentially? stay stable?

$$A = \begin{bmatrix} 0 & 2 & 3 & 2 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0.8 & 0.8 & 0.9 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1.2 & 1.3 & 1.4 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix}$$

solution to 1 (i) : Since the initial population is the same we have

$$\mathbf{n}_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

If L is the Leslie matrix then the required quantity is the top component of

$$L^3 \, \mathbf{n_0}$$

In the language of Maple it is (after you typed `with(linalg)`)

```
evalm(L**3 &* n0)[1,1];
```

For A we have:

```
A:=matrix([[0,2,3,2],[0.4,0,0,0],[0,0.3,0,0],[0,0,0.3,0]]);
```

```
evalm(A**3&*n0)[1,1];
```

gives 17.52, so for the A population they were expected to be 17.52 0-year-olds after 3 years.

For B we have:

```
B:=matrix([[0,0.8,0.8,0.9],[0.6,0,0,0],[0,0.8,0,0],[0,0,0.9,0]]);
```

```
evalm(B**3&*n0)[1,1];
```

gives 5.328, so for the B population they were expected to be 5.328 0-year-olds after 3 years.

For C we have:

```
C:=matrix([[0,1.2,1.3,1.4],[0.4,0,0,0],[0,0.3,0,0],[0,0,0.3,0]]);
```

```
evalm(C**3&*n0)[1,1];
```

gives 6.120, so for the B population they were expected to be 6.120 0-year-olds after 3 years.

Solutions to 1(ii):

You find the eigenvalues of the Leslie matrix, and look at the largest eigenvalue (in absolute value). If it is larger than 1 then the population would grow exponentially, if it is less than 1 it would shrink exponentially and become extinct, if it is exactly 1, then it would last forever and not explode.

For the A population

typing

```
eigenvalues(A);
```

gives:

```
1.09106990125926, -0.643856305759306, -0.223606797749979 + 0.229111841575162*I,

-0.223606797749979 - 0.229111841575162*I
```

so the largest eigenvalue is larger than 1 and the population will explode.

For the B population

typing

```
eigenvalues(B);
```

gives:

```
1.08080122023410, -0.787171750825108, -0.146814734704496 + 0.659878691734053*I,
-0.146814734704496 - 0.659878691734053*I
```

so again, the largest eigenvalue is larger than 1 and the population will explode.

For the C population

typing

```
eigenvalues(C);
```

gives:

```
0.855180083822887, -0.600000000000000, -0.127590041911444 + 0.286261605316927*I,
-0.127590041911444 - 0.286261605316927*I
```

Now the largest eigenvalue in absolute value is **less** than 1, so the population will become extinct.