

Attendance Quiz for Lecture 5 of Dr. Z.'s Dynamical Models in Biology class

1. In a certain species only one-year-olds, two-year-olds, and three-year-olds are fertile. We have

- zero-year-olds can't have babies
- Every 1-year-old female makes 1.5 babies on average
- Every 2-year-old female makes 0.9 babies on average
- Every 3-year-old female makes 0.5 babies on average

We also know

- The probability that a zero-year-old will survive the year is 0.7
- The probability that a one-year-old will survive the year is 0.6
- The probability that a two-year-old will survive the year is 0.4

a. Set up the **Leslie matrix**

Answer to a:

$$L = \begin{bmatrix} 0 & 1.5 & 0.9 & 0.5 \\ 0.7 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{bmatrix}$$

b. If right now there are 10 zero-year-olds, 20 one-year-olds, 5 two-year-olds, and 3 three-year-olds, what is the expected number of 2-year-olds after two years?

Ans. to b

At $t = 0$ the population vector is

$$\mathbf{n}_0 = \begin{bmatrix} 10 \\ 20 \\ 5 \\ 3 \end{bmatrix}.$$

Hence the population vector at time $t = 2$ $\mathbf{n}(2)$ is

$$\begin{aligned} \mathbf{n}(2) &= L^2 \mathbf{n}_0 = \begin{bmatrix} 0 & 1.5 & 0.9 & 0.5 \\ 0.7 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{bmatrix}^2 \begin{bmatrix} 10 \\ 20 \\ 5 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 1.05 & 0.54 & 0.20 & 0 \\ 0 & 1.05 & 0.63 & 0.35 \\ 0.42 & 0 & 0 & 0.0 \\ 0.0 & 0.24 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 5 \\ 3 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 22.30 \\ 25.20 \\ 4.2 \\ 4.8 \end{bmatrix} .$$

Looking at the third component (the number of 2-year-old) we get:

Ans. to b: The expected number of 2-year-olds after two years is 4.8.