**Solutions to Attendance Quiz for Lecture 1 of Dr. Z.'s Dynamical Models in Biology class**

**1.** Compute $x_5$ of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + x_{n-2}$$

subject to the initial conditions

$$x_0 = 1, x_1 = 2 \quad .$$

**Sol. to 1**:

$$x_2 = x_1 + x_0 = 2 + 1 = 3$$

$$x_3 = x_2 + x_1 = 3 + 2 = 5$$

$$x_4 = x_3 + x_2 = 5 + 3 = 8 \quad .$$

$$x_5 = x_4 + x_3 = 8 + 5 = 13 \quad .$$

**Ans. to 1**: $x_5 = 13$.

**2.** Solve explicitly the recurrence equation

$$x_n = 2x_{n-1} - x_{n-2} \quad ,$$

with initial conditions

$$x_0 = 0, x_1 = 2 \quad .$$

**Sol. of 2**: The **characteristic equation** is

$$z^2 = 2z - 1 \quad ,$$

In other words

$$z^2 - 2z + 1 = 0 \quad .$$

Solving

$$(z-1)^2 = 0 \quad .$$

There is just one **double root**, $z = 1$. So the template

$$x_n = C_1 + C_2 n \quad .$$

where $C_1$ and $C_2$ are yet to be determined.

When $n = 0$ we have

$$0 = x_0 = C_1 + C_2 \cdot 0 \quad ,$$

so

$$C_1 = 0 \quad .$$

When $n = 1$ we have

$$2 = x_1 \ = \ C_1 + C_2 \cdot 1 \quad .$$

So $C_2 = 2$

**Ans. to 2**: $x_n \ = \ 2n$.

**Comment**: this problem was a bit confusing since the double root is $z = 1$ and $1^n = 1$ so because the answer was so simple, it was challenging.