Solutions to Attendance Quiz for Lecture 17 of Dr. Z.'s Dynamical Models in Biology class

Name: Dr. Z.

1. Find all the **stable equilibrium points** of the 2D *continuous* dynamical system given by the differential equations

$$\frac{dx}{dt} = 1 - \frac{2x}{1+y} \quad ,$$
$$\frac{dy}{dt} = 1 - \frac{2y}{1+x} \quad .$$

Sol. of 1: To find the equilibrium points we have to solve

$$1 - \frac{2x}{1+y} = 0 \quad ,$$
$$1 - \frac{2y}{1+x} = 0 \quad .$$

Simplifying (you do it!) you get a system of two equations with unknowns x and y, whose solution is

$$(x, y) = (1, 1) \quad .$$

So the set of equilibrium points is $\{(1,1)\}$.

Next we take the Jacobian of the transformation from R^2 to R^2

$$(x, y) \rightarrow (1 - \frac{2x}{1+y}, 1 - \frac{2y}{1+x}) \quad .$$

$$J(x, y) = \begin{bmatrix} -frac21+y & \frac{2x}{(1+y)^2} \\ \frac{2y}{(1+x)^2} & -frac21+x \end{bmatrix}$$

Plugging-in $x = 1$, $y = 1$, we get

$$J(1, 1) = \begin{bmatrix} -frac21+1 & \frac{2 \cdot 1}{(1+1)^2} \\ \frac{2 \cdot 1}{(1+1)^2} & -frac21+1 \end{bmatrix}$$
$$\begin{bmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix} \quad .$$

The characteristic equation is:

$$(1 + \lambda)^2 - \frac{1}{4} = 0$$

$$(1+\lambda)^2 - (\frac{1}{2})^2 = 0$$

$$(\frac{1}{2}+\lambda)(\frac{3}{2}+\lambda) = 0$$

The eigenvalues are $-\frac{1}{2}$ and $-\frac{3}{2}$ since they are both negative (in particular, they have a negative real parts) this is **stable**.

Ans.: So the set of stable equilibrium points is $\{(1,1)\}$.