

Attendance Quiz for Lecture 15 of Dr. Z.'s Dynamical Models in Biology class

Name: (print!)

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1. Recall that the Hardy-Weinberg rule tells you that if in the current generation the frequency of genotypes AA , Aa and aa are u , v , and $1 - u - v$, respectively, then in the next generation (u, v) transforms according to:

$$(u, v) \rightarrow \left(u^2 + vu + \frac{1}{4}v^2, -2uv - 2u^2 + 2u - \frac{1}{2}v^2 + v \right)$$

If right now, $\frac{1}{2}$ of the population have genotype AA , $\frac{1}{4}$ of the population have genotype Aa , what is the fraction of aa genotypes (i) Right now? (ii) In the next generation? (iii) In ten generations?

Sol. to 1(i) This is easy, since the probabilities have to add-up to 1, the fraction of aa genotype is $1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$.

Ans. to 1(i): The fraction of the population with genotype aa right now is $\frac{1}{4}$.

Sol. of 1(ii)

$$\left(\frac{1}{2}, \frac{1}{4}\right) \rightarrow \left(\left(\frac{1}{2}\right)^2 + \frac{1}{2} + \frac{1}{4}\left(\frac{1}{4}\right)^2, -2\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) - 2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - \frac{1}{2}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\right) = \left(\frac{25}{64}, \frac{15}{32}\right) \quad .$$

Hence the fraction of the population with genotype aa after one generation is

$$1 - \frac{25}{64} - \frac{15}{32} = \frac{9}{64} \quad .$$

Ans. to 1(ii): The fraction of the population with genotype aa after one generation is $\frac{9}{64}$.

Sol. of 1(iii): You can apply the Hardy-Weinberg transformation to $(\frac{25}{64}, \frac{15}{32})$ nine more times, but that would be stupid. After one generation it stays the same for ever after!

Ans. to 1(iii): The fraction of the population with genotype aa after ten generations is $\frac{9}{64}$.