

Homework for Lecture 9 of Dr. Z.'s Dynamical Models in Biology class

Version of Oct. 5, 2025 (Correcting a typo, found by Rachel Adelmam, who won a dollar).

Email the answers (as a .pdf file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Oct. 6, 2025.

Subject: hw9

with an attachment hw9FirstLast.pdf

1. Solve the boundary value problem

$$a(n+2) = 5a(n+1) - 6a(n) = 0 \quad ; \quad a(n) = 1, \quad a(L) = 2 \quad .$$

2. (Use a calculator or Maple) In a gambler's ruin problem you currently have 700 dollars and the maximum amount is 1000.

(i) What is the probability of exiting a winner if your prob. of winning a dollar is 0.5 (and losing a dollar is 0.5)

(i') What is the expected number of rounds until you exit either a winner or loser?

(ii) What is the prob. of exiting a winner if your prob. of winning a dollar is 0.49 ?

(iii) What is the prob. of exiting a winner if your prob. of winning a dollar is 0.499 ?

3. Derive, all by yourself, the formula for the prob. of exiting a winner in a gambler's ruin where the prob. of winning a dollar is p if the max. amount is L and you currently have n dollars.

4. Prove that the expected duration of staying in a fair casino with max. amount is L and you currently have n dollars is $n(L - n)$.