

Homework for Lecture 4 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Sept. 22, 2025.

Subject: hw4

with an attachment hw4FirstLast.pdf and/or hw4FirstLast.txt

1. Give an example of a second-order linear differential equation, with two specific functions that are solutions and verify that their sum also satisfies that same differential equation.
2. Verify that $y_1(t) = t^2$ satisfies the differential equation

$$y'(t)^2 - 4y(t) = 0 \quad .$$

Would you expect the function $y_2(t) = 2y_1(t) = 2t^2$ to also be a solution? (after all it is a constant multiple of $y_1(t)$). Explain. Verify that indeed $y_2(t)$ is **not** a solution.

3. Give an example of a second-order linear recurrence equation, with two specific sequences that are solutions and verify that their sum also satisfies that same recurrence equation.
4. Consider the non-linear recurrence

$$a(n) = a(n-1)^2 \quad , n \geq 0 \quad .$$

Check that both sequences

$$a_1(n) := 2^{2^n} \quad , \quad a_2(n) := 3^{2^n} \quad ,$$

are solutions. Does it follow that the new sequence

$$a_3(n) := a_1(n) + a_2(n) = 2^{2^n} + 3^{2^n} \quad ,$$

is automatically yet-another-solution? Explain why or why not. By directly plugging-in into the recurrence find out whether it is true.

5. Write the Maple commands to solve each of the following problems, and give the Maple output.
 - a. Solve the Initial Value Problem Differential Equation

$$y''(x) + y(x) = 0 \quad , \quad y(0) = 1 \quad , y'(0) = 1 \quad .$$

- . **b** Solve the Initial Value Problem Difference Equation

$$a(n) - 3a(n-1) + a(n-2) = n \quad a(0) = 1, a(1) = 3 \quad .$$

- c.** Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix}$$