

Homework for Lecture 12 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (as .pdf file) to

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by 8:00pm Monday, Oct. 20, 2025.

Subject: hw12

with an attachment hw8FirstLast.pdf and/or hw12FirstLast.txt

Using

<http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt>

1. What is the ultimate period of

`evalf(Orb((1-x)*(2-x)*(3-x)/12,x,0.5,1000,1010),10);?`

By Playing 'high-low', estimate the cut-offs r_1, r_2, r_3, r_4 such that

the recurrence

$$a(n+1) = (1 - a(n))(2 - a(n))(3 - a(n))/r \quad ,$$

has

- ultimate period 1 for $r_1 < r$
- ultimate period 2 for $r_2 < r < r_1$
- ultimate period 4 for $r_3 < r < r_2$

(challenging!):

- ultimate period 8 for $r_4 < r < r_3$

2.

In the Maple package <http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt>

Run the following line 20 times

```
T:=RT([x,y],30); SSg(T,[x,y]); ORB(T,[x,y,z],[100., 200.],1000,1020);
```

Here the first command, `T:=RT([x,y],30);` generates a random rational transformation in R^2 , the second command, `SSg(T,[x,y]);`, finds steady-states (i.e. fixed points of the transformation T , using

Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T, [x,y], [100., 200.], 1000, 1020)`; finds the long-term behavior.

If all goes well, the ultimate behavior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

```
ORB(T, [x,y], [100., 200.], 1000, 1020)[-1];
```

was not part of the output of

```
SSg(T, [x,y]);
```

3.

In the Maple package <http://sites.math.rutgers.edu/~zeilberg/Bio25/DMB12.txt>

Run the following line 20 times

```
T:=RT([x,y,z], 10); SSg(T, [x,y,z]); ORB(T, [x,y,z], [100., 200., 300.], 1000, 1020);
```

Here the first command, `T:=RT([x,y,z], 10)`; generates a random rational transformation in R^3 , the second command, `SSg(T, [x,y,z])`; finds steady-states (i.e. fixed points of the transformation `T`, using Maple's `solve` command, but unfortunately not all, and the last command, `ORB(T, [x,y,z], [100., 200., 300.], 1000, 1020)`; finds the long-term behavior.

If all goes well, the ultimate behavior of the orbit should be ONE of the outputs of `SSg`, but sometimes it misses it. Out of the 20 tries, list all the cases where

```
ORB(T, [x,y,z], [100., 200., 300.], 1000, 1020)[-1];
```

was not part of the output of

```
SSg(T, [x,y,z]);
```