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ANALYSIS

Harvesting and conserving a species when numbers are low: population viability and gambler's ruin in bioeconomic models

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Abstract

In bioeconomic models of renewable resources, population viability is either ignored entirely or the minimum viable population (MVP) is considered a crisp threshold below which a species is driven to extinction. Neither is consistent with ecological science. The purpose in this paper is, firstly, to enhance ecological realism in economic models by incorporating insights regarding population viability from conservation biology. We consider the effects of chance on optimal management and briefly discuss key results regarding population viability as derived by biologists. Second, as a means to 'balance interests and morality', we suggest a fuzzy compromise between the economist's and the biologist's preferred stock sizes.

Keywords: Minimum viable population, MVP; Extinction; Uncertainty; Stochasticity; Renewable resource management; Fuzzy multiple objective decision making

1. Introduction

The World Fishery Trust is concerned with the loss of genetically unique salmon stocks on Canada's West Coast. Although the fish are managed as a common resource, mismanagement continues to threaten the fishery because of such features as subsidies to commercial fishermen, politics associated with the native fishery and a salmon war with the United States. In addition, the recent El Nino phenomenon has pushed mackerel further north than normal; mackerel are ferocious feeders of young salmon, thereby threatening certain stocks on the west coast of Vancouver Island. Seals, killer whales and other species also feed on the salmon, while salmon are often caught as by-catch when fishing for other species.

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Combined with the unpredictability of the long journeys to spawning grounds, and the fact that each river has unique salmon stocks associated with it, it is clear that many factors contribute to the viability of salmon stocks. Clearly, management of salmon stocks occurs in an atmosphere of uncertainty. This holds for management of most, if not all, renewable resources and endangered species.

Loss of a species' (or unique stock's) viability, or even extinction, can be a deliberate or an inadvertent event.

> Commercially important species are often overharvested to economic depletion (many fisheries and forest dwelling species), to near extinction (the blue whale, right whale, northern elephant seal, American bison, black rhino, white rhino, for example) or to extinction (Stellar's sea cow, great auk, Caribbean monk seal). Over-exploitation, usually combined with habitat destruction and/or introduced species threatens about one-third of the endangered mammals and birds of the world (Lande et al., 1994, p. 88).

Commercial exploitation of a resource, and its potential extinction, are generally investigated using simple deterministic growth functions (Clark, 1973, 1990; Cropper, 1988; Swanson, 1994; Farrow, 1995 Schulz, 1996; for exceptions, see Reed, 1979; Pindyck, 1984). However, as noted for salmon, extinction of a species may be (and most often *will be*) caused by stochastic perturbations, rather than predictable and controllable systematic pressures such as hunting and habitat destruction. While maintaining a (small) population of a species may be an attractive option in bioeconomic models, survival of such populations is not guaranteed due to the vagaries of nature.

In most bioeconomic models of renewable resource exploitation, mathematical tractability is preferred over ecological realism, with many or most ecological processes poorly represented. For example, most models consider species (or pairs of species) in isolation from other species in the ecosystem, while the biophysical environment is typically considered to be unchanging over time. Notwithstanding the benefits of working with (greatly) simplified models, outcomes and recommendations from such models should be treated with extreme caution as the underlying mechanisms of population growth and decline are quite different in reality. The risk is that 'optimal management' that ignores inherent, uncontrollable stochasticity will potentially result in extinction, which raises a set of intergenerational and moral issues that lie outside bioeconomic models.

In this paper, we incorporate somewhat greater biological realism in bioeconomic models by considering the concept of *minimum viable population* (MVP) more carefully. Although the MVP concept is a focal point in conservation biology and is well known in natural resource economics (e.g. Clark, 1990; Conrad and Clark, 1987), it is often ignored in bioeconomic applications where the growth function is generally assumed to be strictly concave in stock size. Because of various (uncontrollable) stochastic elements, small numbers associated with MVP are a high-risk gamble. Thus, management based on optimization models that permit (encourage) holding of small stocks is likely to lead to extinction, a phenomenon referred to as the Gambler's Ruin of bioeconomic modeling.

We approach our task by first considering the notion of minimum viable populations in bioeconomic models and extinction in greater detail in the next section. Then, in Section 3, we examine ecological factors leading to extinction, outside of those controlled by humans (at least in the context of bioeconomic models). In Section 4, we develop a bioeconomic model of exploitation that includes existence values that increase with stock size. Population growth is taken to be stochastic because of randomness in food availability and the potential for catastrophe. We demonstrate that, despite non-use values that offset a proclivity to harvest the species to low population levels, the species can go extinct because of uncontrollable stochastic factors. In light of the potential for Gambler's Ruin, the idea of a minimum viable population needs to be reconsidered, which we do in Section 5. Further, in contrast to standard bioeconomic models where a social planner maximizes a discounted net social benefits function, some form of collective choice rule is required to balance fairly economic efficiency against the biological ethic of preventing any species from going extinct. We suggest, in Section 6, a means for doing this by employing insights from fuzzy set theory. Our conclusions ensue.

2. Bioeconomic models and extinction

Extinction of plant and animal species in general (and the threat of extinction of certain high-profile animal species in particular) is a problem that biologists have brought to the attention of policy makers. Swanson (1994) argues that the economics of biodiversity loss, or the narrowing of the gene pool through the loss of many (unknown) species, and extinction of known species by overexploitation are essentially the same general problem.

> Human societies must select a portfolio of assets from which they derive a flow of benefits, and one important part of this portfolio is the range of biological assets upon which we depend.… Given capital constraints it will sometimes be optimal to disinvest in one asset and invest the receipts in another asset; that is, it will be socially optimal to engage in conversions between assets to equilibrate returns.... The fundamental force driving species decline is always the relative rate of investment by the human species. It is the human choice of another asset over a biological asset that results in the inevitable decline of that species (p. 805).

In an efficient steady state, the resource should be maintained at such a level (stock size) that, at the margin, the return from that asset is equated with the return from other assets in the economy — i.e., the discount rate r. The return from investing in the biological asset typically depends on the marginal growth of the asset $G'(x)(=\partial G/\partial x)$, the marginal non-use values of the in situ stock, and the stock dependent harvest costs.

> Slow growth relative to other assets in the economy is in and of itself a route to species extinction. Resources, even biological resources, must be competitive as productive assets if there is to be a force for their retention in a world of scarce resources (Swanson, 1994, p. 807).

The implication is that from an economic perspective biological assets should be removed from the human portfolio when their rate of return falls below competitive levels, when they represent 'inferior investments'.

Economists generally employ the well-known logistic growth function, $G(x)=\gamma x(1-x/K)$, in bioeconomic models of resource management. Here γ is the intrinsic growth rate, x is in situ stock size and K is the carrying capacity of the natural environment. When the stock is small, growth will be modest, even under the most favorable conditions. In terms of the fishery, the reason may be that there are few female fish to produce offspring. Growth of the stock will also be small when the population is close to its carrying capacity or maximum size — when it has filled its niche in the ecosystem, and the ecosystem is unable to support further growth. Possible reasons are food scarcity or spreading of diseases because of high population densities.

A more complete, but still overly simplified growth function includes the MVP as a constant and crisp number, or threshold. This corresponds with the known and constant population level below which, without intervention, the population decreases and eventually approaches zero. The rationale for a minimum survival stock may be based on the difficulty of individuals to find suitable mates for reproduction, or as a defense against predators. Neither the probabilities of reproduction or predation are explicitly modeled. Incorporating MVP in a growth function amounts to allowing for a (deterministic) non-concave interval of negative growth at low stock densities, so that the growth curve, *G(x)*, intersects the horizontal axis at MVP (Clark, 1990 Bulte and van Kooten, 1999a). A possible specification of the growth function that allows for such an interval is:

$$G(x)=\gamma x\left(\frac{x}{\mathrm{MVP}}-1\right)\left(1-\frac{x}{K}\right)$$

Growth functions are *purely compensatory* if $\mathrm{MVP}=0$ and $G(x)$ is strictly concave (such as the logistic growth function); *depensatory* if $G(x)$ is initially convex and later concave; and, finally, said to exhibit *critical depensation* if $\mathrm{MVP}>0$ and $G(x)$ is initially convex and later concave (Clark, 1990). The carrying capacity K is a stable equilibrium whereas MVP, the minimum viable population size, would be unstable; that is, when the actual population deviates from the equilibrium population, the equilibrium population cannot be restored without intervention, as $G(x)<(>)0$ when $x<(>)\mathrm{MVP}$.

Due to an assumed concave growth function, the relative rate of return to (many) biological assets is not fixed but can be manipulated: reducing the stock x over the concave segment of the growth curve implies increasing $G'(x)$. Hence, to ensure a high (competitive) rate of return to in situ stocks of biological assets at the margin, economic theory states that stocks should be kept at "low" levels (if certain conditions with respect to harvesting costs are satisfied). In a deterministic world, this does not pose any particular problem, as long as the economically-optimal stock x^* exceeds the population viability threshold or MVP (which may or may not be the case). However, the concept of population viability and survival is more complex than usually modeled by economists. For a good appreciation of extinction and viability issues it is necessary to expand the ecological underpinnings of the economic models.

3. Ecological considerations and extinction

The economist's deterministic view of the world is not consistent with the way ecologists view extinction. In addition to controllable and predictable processes such as hunting and habitat conversion (so-called "systematic pressures") that are the main concern of economists, ecologists appreciate that species viability is determined by stochastic perturbations. The latter elude human prediction and control because they are or appear random, causing uncertainty. While small populations clearly run a greater risk of extinction than large populations, the viability concept should be cast in terms of chance.[1] To be more specific, maintaining low populations of biological resources as "competitive assets" may be risky for four, not necessarily mutually-exclusive reasons (Primack, 1998; Quammen, 1996; Soulé, 1987).

1. *Demographic stochasticity*. Field studies indicate that for many species there are large accidental variations in birth rates, death rates and the sex ratio (the last five surviving individuals of the now extinct dusky seaside sparrow were all males). As long as the population is large, average rates may provide an accurate description of the population. For populations smaller than, say, 50 individuals, individual variation in birth and death rates cause the population size to fluctuate randomly up or down (Primack, 1998). Due to the Gambler's Ruin property of such processes, which emphasizes that the zero stock serves as an absorbing boundary (Raup, 1991), such fluctuating populations have a high probability of going extinct. For some species the problem may be worsened by social dysfunction; it has been observed that the social structure of populations (defense against predators, finding food and mates, etc.) may fall apart below a certain threshold level (the so-called Allee effect).
2. *Environmental stochasticity*. Random variations in the biological and physical environment may drive fluctuations in species abundance, affecting all individuals in the population. For example, variations in weather, food supply, predators, parasites and competitors are major determinants of a species' ability to survive, adding to the Gambler's Ruin property.

[1] As argued by Soulé (1987) "... there are not hopeless cases, only expensive cases and people without hope" (p. 181). Examples of apparently hopeless cases are the white rhinoceros (of which no more than 20 animals were alive in the 1920s) and the Mauritius kestrel (of which no more than two pairs were known to exist in the wild in the mid-1970s). With human intervention, both species have made a reasonably successful come back, although future viability is by no means assured.

3. *Catastrophes*. In some senses, species conservation resembles flood control. It is not enough to assure protection (survival) in 'normal years' (taking normal environmental fluctuations into account). Rather, it is important to assure against the possibility of exceptionally harsh circumstances, against, for example, the occurrence of a 'once-in-100-years' flood, storm, earthquake, drought and/or extreme fire, where as much as 70–90% of populations may be killed at once.
4. *Genetic stochasticity*. To adapt to changing circumstances, a population should have sufficient genetic variability. Individuals of a species have different 'forms' of the same gene (know as alleles), and the frequency of different alleles may range from common to rare. Some alleles are useful, either now or in the future, while others are potentially harmful. Allele frequencies may change over time for a number of reasons. Small populations are susceptible to change because of chance. Two harmful genetic processes are at work in small populations. First, helpful alleles may become more rare and eventually disappear due to the random process of genetic drift. Low frequency alleles simply disappear, which is just another example of the Gambler's Ruin effect. While migration and mutation may replace the genetic information that is lost, these processes are insignificant compared to genetic drift in populations of 100 individuals or less (Primack, 1998). Second, in normal populations individuals do not normally mate with their close relatives, but the mechanisms that prevent such inbreeding fail when population size is small. Mating among close relatives may result in inbreeding depression, a process through which harmful alleles manifest themselves (are expressed as homozygotes rather than heterozygotes). The extent to which inbreeding may cause problems depends on the 'genetic load', or the sum of harmful recessive alleles within the population. Populations with a large genetic load that suffer a large reduction in size are likely to suffer from harmful effects on inbreeding depression.

Demographic and environmental stochasticity are unlikely to wipe out large populations of an animal species, while small populations are sensitive to small demographic and/or environmental "shocks" (e.g., if by chance all four Mauritius kestrels remaining in the 1970s would have been males, the species would have gone extinct as did the dusky seaside sparrow).[2] Also, for obvious reasons, large populations are safer from genetic drift and inbreeding depression than small populations. The point is that species viability cannot be modeled by a simple threshold, but rather as a continuum with the degree of safety of a population monotonically increasing in stock size x.

Introducing stochasticity has far-reaching impacts on extinction probabilities. This is demonstrated by Lande et al. (1994), who analyze extinction risk in fluctuating populations. In a numerical model, they allow for variation in population growth due to demographic and environmental stochasticity and find that incorporating stochasticity into optimal harvesting strategies results in faster extinction of the target population (see also May, 1994). Due to environmental and demographic stochasticity, eventual extinction appears inevitable for most populations, with the fossil record suggesting that the average life span of a species is about 2–4 million years (Leakey and Lewin, 1995; Raup, 1991). From this perspective, conservation of species may merely imply delay of the inevitable. However, the average time to extinction is sensitive to the harvesting regime, and postponing extinction can be considered the aim of conservationists.

4. A stochastic model of population viability

In this section, we develop a simple model that captures catastrophes and demographic and environmental stochasticity. Assume that, ex ante,

[2] The probability that N individuals are of the same sex (be it male or female) is given by $2(1/2)^N$. For the four remaining Mauritius kestrels, this implies that the probability of extinction due to an unfavorable sex ratio was equal to 1/8. For the 20 white rhinos, the probability falls to 0.000002. For 500 individuals, the chance drops essentially to zero!

society is risk neutral and seeks to maximize the use and non-use benefits of conserving a marine species (i.e., we ignore the opportunity cost of habitat, although this could easily be included in the analysis). The problem can be represented as:

$$\text{Maximize}_h \ [B(h, x) - c(h, x)]e^{-r^t}dt \tag{1}$$

where $B(h, x)$ are benefits from harvesting h units of the species ($\partial B/\partial h > 0$) and from non-use value associated with conservation of the in situ stock x or population abundance ($\partial B/\partial x > 0$);[3] $c(h,x)$ are harvest costs as a function of harvest amount and existing stocks ($\partial c/\partial h > 0$, $\partial c/\partial x < 0$); and r is the (social) discount rate. Maximization takes place subject to the following stochastic processes:

$$\mathrm{d}x = [G(x, f) - h]\mathrm{d}t + \sigma_1(x)\mathrm{d}w_1 - j(x)\mathrm{d}q \tag{2}$$

$$\mathrm{d}f = \sigma_2(f)\mathrm{d}w_2. \tag{3}$$

In Eq. (2), it is assumed that expected growth G of the resource is a function of current stock size x and stochastic food (or prey) availability f.[4] Since f is treated as a random variable (see below), environmental stochasticity is included in the model through the impact of food availability on (net) regeneration.

In the absence of catastrophes, the expected change in species abundance over the period $\mathrm{d}t$ is $G(x, f) - h$. The term $\sigma_1(x)\ \mathrm{d}w_1$ on the RHS of Eq. (2) represents random disturbances in the stock due to demographic variation. The term $\mathrm{d}w_1$ is an increment of the stochastic, Wiener process w_1 (with Brownian motion), such that $\mathrm{d}w_1 = \varepsilon_1(t)$, where $\varepsilon_1(t)$ is a serially uncorrelated and normally distributed random variable with zero mean and unit variance (Pindyck, 1984; Dixit and Pindyck, 1994). Assume $\partial\sigma_1/\partial x \geq 0$ and $\sigma_1(0) = 0$.

The term $j(x)\ \mathrm{d}q$ describes the disruptive effect of catastrophes on population size. Catastrophes cause infrequent but discrete changes (or jumps) in species abundance. Assume that catastrophes can be modeled as a Poisson process. Denote by α the mean arrival rate of a catastrophe, such that the probability of occurrence of such an event over the time period $\mathrm{d}t$ is given by $\alpha\ \mathrm{d}t$; the probability of no catastrophe is simply $(1 - \alpha\mathrm{d}t)$. Now, $\mathrm{d}q = 0$ with probability $(1 - \alpha\mathrm{d}t)$, and $\mathrm{d}q = \beta$ with probability $\alpha\ \mathrm{d}t$, such that $\beta j(x)$ is the downward adjustment (jump) in species abundance after a catastrophe has occurred (Dixit and Pindyck, 1994).

The stochastic process in Eq. (3) describes food availability over time, which is simply assumed to be a function of a random component (e.g., weather fluctuations), and w_2 is a Wiener process.

We assume $E(\mathrm{d}w_1\,\mathrm{d}q) = E(\mathrm{d}w_2\,\mathrm{d}q) = 0$, and that $j(x) = x$, so that, if a catastrophe occurs, q falls by some fixed percentage β $(0 \leq \beta \leq 1)$ so that $(1 - \beta)$ times the initial population remains after an event. This implies that $\mathrm{d}x = [G(x, f) - h]\mathrm{d}t + \sigma_1$, with probability $\frac{1}{2}(1 - \alpha\ \mathrm{d}t)$; $\mathrm{d}x = [G(x, f) - h]\mathrm{d}t - \sigma_1$ with probability $1/2(1 - \alpha\mathrm{d}t)$; and $\mathrm{d}x = -\beta x$ with probability $\alpha\ \mathrm{d}t$.[5] Hence, the expected change in x over time, $(1/\mathrm{d}t)\mathrm{E}(\mathrm{d}x)$, is defined as $[G(x, f) - h](1 - \alpha\mathrm{d}t) - \alpha\beta x$. Define ρ as the correlation coefficient between $\mathrm{d}w_1$ and $\mathrm{d}w_2$. Note that ρ is also the covariance per $\mathrm{d}t$ for $\mathrm{d}w_1$ and $\mathrm{d}w_2$ since the standard deviation per unit of time for these processes is equal to one.

By Ito's lemma, dynamic programming can be used to solve this problem (Kamien and Schwarz, 1994; Dixit and Pindyck, 1994). As $E(\mathrm{d}w_1\,\mathrm{d}w_2) = \rho\,\mathrm{d}t$, Bellman's fundamental equation of optimality can be written as:

$$\begin{aligned} &rV(x, f) \\ &= \max_h \ \{B(h, x) - c(h, x) + [G(x, f) - h]V_x \\ &\quad + 1/2\sigma_1^2 V_{xx} + 1/2\sigma_2^2 V_{ff} + \sigma_1\sigma_2\rho V_{xf} \\ &\quad - \alpha[V(x, f) - V((1-\beta)x, f)]\}, \end{aligned} \tag{4}$$

where $V(x, f)$ is the optimal value function. An optimal solution requires that $\partial B/\partial h - \partial c/\partial h - V_x = 0$, which implies that the shadow price of the

[3] We model non-use values as a function of species abundance, but these should be separated into two components, one dealing with survival of the species (ensuring minimum viable population) and the other related to numbers in excess of MVP (see Bulte and van Kooten, 1999a).

[4] Alternatively, we could model additional interactions with the environment by including, for example, predators or pests p, such that $\partial G/\partial p < 0$. For simplicity, we do not pursue this further.

[5] With catastrophe, there is no harvest or normal growth in the stock.

renewable resource (V_x) should be equal to the marginal benefit from harvesting the species. Substituting the optimal harvest level h^* in Eq. (4) and differentiating with respect to x gives:

$$
\begin{aligned}
rV_x = {} & (\partial B/\partial h - \partial c/\partial h - V_x)\partial h/\partial x - \partial c/\partial x \\
& + \partial B/\partial x \ + \partial G/\partial x \ V_x + (G-h)V_{xx} \\
& + \tfrac{1}{2}\sigma_1^2 V_{xxx} + (\partial\sigma_1/\partial x)V_{xx}\sigma_1 \\
& + (\partial\sigma_1/\partial x)V_{xf}\sigma_2\rho + \sigma_1\sigma_2\rho V_{xxf} + \tfrac{1}{2}\sigma_2^2 V_{xff} \\
& - \alpha\beta V_x, \qquad (5)
\end{aligned}
$$

which is evaluated at h^*.

It is possible to simplify Eq. (5). Taking a second-order Taylor series expansion of V(x, f) and differentiating with respect to x gives:

$$
\begin{aligned}
\mathrm{d}V_x = {} & V_{xx}\mathrm{d}x + V_{xf}\mathrm{d}f + 1/2V_{xxx}\mathrm{d}x^2 + 1/2V_{xff}\mathrm{d}f^2 \\
& + V_{xxf}\mathrm{d}x\mathrm{d}f, \qquad (6)
\end{aligned}
$$

which is readily rewritten as:

$$
\begin{aligned}
\mathrm{d}V_x = {} & V_{xx}[(G(x,\ f) - h)\mathrm{d}t + \sigma_1\mathrm{d}w_1 - x\mathrm{d}q] \\
& + V_{xf}\sigma_2\mathrm{d}w_2 + 1/2\sigma_2^2 V_{xff}\mathrm{d}t \\
& + 1/2V_{xxx}(\sigma_1^2\mathrm{d}t + \alpha\beta^2x^2\mathrm{d}t) + V_{xxf}\sigma_1\sigma_2\rho\,\mathrm{d}t \qquad (7)
\end{aligned}
$$

In deriving Eq. (7), we substituted Eq. (2) and Eq. (3) for dx and df, respectively, and used $(\mathrm{dw_i})^2 = \mathrm{dt}$, $\mathrm{d}w_1 dw_2 = \rho$, $(\mathrm{d}t)^2 = (\mathrm{d}t)^{3/2} = \mathrm{d}w_i\mathrm{d}q = 0$. Taking the expectation of Eq. (7), noting that $E(\mathrm{d}w_i) = 0$, and dividing by dt provides the expected rate of change in the marginal value of the renewable resource $(1/\mathrm{d}t)E(\mathrm{d}V_x)$. Then we substitute this result into Eq. (5), noting that, for an optimum solution, $\partial B/\partial h - \partial c/\partial h - V_x = 0$ holds. The optimal in situ stock of the renewable resource is then implicit in the following condition:

$$
\begin{aligned}
& r + \alpha\beta - \frac{1}{V_x}\{[(\partial\sigma_1/\partial x)\sigma_1 + \alpha\beta x]V_{xx} - \frac{1}{2}V_{xxx}\alpha\beta^2x^2 \\
& + (\partial\sigma_1/\partial x)\sigma_2\rho V_{xf}\} \\
& = \partial G/\partial x + \frac{1}{V_x}\{(1/\mathrm{d}t)E(\mathrm{d}V_x) + (\partial B/\partial x) - \partial c/\partial x\}. \qquad (8)
\end{aligned}
$$

This condition is an extended version of the stochastic modified golden rule derived by Pindyck (1984), Olsen and Shortle (1996), Bulte and van Kooten (1999b). It states that, at the margin, the resource owner (society) is indifferent between harvesting the resource and investing the proceeds elsewhere in the economy (LHS) and holding the resource in situ (RHS).[6]The marginal benefit from conserving a unit, or the expected rate of return, consists of (i) the effect on resource growth, (ii) the expected capital gain, (iii) marginal non-use values, and (iv) the depressing effect of increasing stock size on harvest cost. Since $c(h, x)$, $B(h, x)$ and $G(x, f)$ are likely nonlinear in x, they are affected by stochastic fluctuations in x, even though the expected values of these disturbances equal zero. This is due to Jensen's inequality. Stochastic fluctuations reduce the expected growth rate, as $G(x, f)$ is concave in x. This increases scarcity and reduces optimal harvest levels, and increases expected catch costs, thus creating an incentive to increase harvesting to reduce future cost increments. This indicates that the effect of (random) changes on optimal harvest policies is analytically ambiguous.

The LHS of Eq. (8) describes the social opportunity cost of conservation, which is more complex. The first term is the social opportunity cost of capital, augmented by a term that comes from the Poisson disturbance. It is well known that if a benefit stream is interrupted as a result of a Poisson event (with arrival rate α), the expected present value of the stream of benefits can be calculated as if it had never ceased, but must be added to the discount rate (Dixit and Pindyck, 1994, p. 87). In this specific case, however, the species is not terminated entirely after a random "event", but is merely reduced in abundance. This explains why we adjust the arrival rate downward (i.e., multiply α by $\beta \leq 1$). Nevertheless, the profitability of investing in conservation of the in situ stock is reduced, and this term provides an incentive to reduce the optimal stock.

The second term on the LHS is Pindyck's "risk premium" (Pindyck, 1984, p. 294), adjusted for the possibility of jumps, with the adjustment being $\alpha\beta xV_{xx}/V_x$. Pindyck's original risk premium is

[6] This is clear after $(B_{h}-c_h)$ is substituted for V_x, and $(1/\mathrm{d}t)E[\mathrm{d}(B_h - c_h)]$ for $(1/\mathrm{d}t)E[\mathrm{d}V_x]$.

the increase in stock growth variance attributable to the marginal in situ unit multiplied by an implicit index of absolute risk aversion ($-V_{xx}/V_x$). However, the possibility of a stock-dependent catastrophe further unambiguously increases the (expected) rate of return that is demanded by a resource owner to conserve the marginal in situ unit.

The third term is a correction for non-marginal changes in stock size due to catastrophes. Since the curvature of $V(x, f)$ is not necessarily constant over the range of values that x can take, such non-marginal changes will affect the level of absolute risk aversion (for constant curvature, $V_{xxx}=0$). Depending on whether $-V_{xx}/V_x$ increases or decreases as x falls (i.e., on the shape of $V(x, f)$), this implies an incentive to decrease or increase in situ stock levels, respectively.

The fourth term on the LHS is identical to Bulte and van Kooten's (1999b) adjustment to the expected rate of return required to hold the marginal unit in situ. The sign of this term depends on V_{xf} and ρ, and is analytically ambiguous. When $\rho V_{xf} > (<) 0$, the adjustment term is negative (positive); hence, the required rate of return decreases (increases), so the adjustment represents an incentive to increase (decrease) the optimal stock.

Even though we have included non-use values as an argument in the model, species survival (population viability in the long run) is by no means assured. For example, when the opportunity cost of capital or the probability of catastrophe is high, the economically-optimal stock size may well be so low that, in the short or medium term, the stock is driven to extinction because of chance effects. Indeed, it may even be economically optimal to harvest the very last individual of the species, although rising harvest costs and marginal non-use values probably prevent this from happening.

5. Viability requirements and allowable harvesting

The foregoing model is based on the assumption that a manager maximizes net (social) benefits. With the aid of this model, it is possible to consider population viability of the species at the preferred stock size. The minimum-viable population is usually considered more restrictive, and used in a different context. More specifically, the use of MVPs has been advocated in the context of sustainable management. Assume that, based on ethical considerations, minimum viability requirements are imposed as constraints on harvest decisions. This implies abandoning the optimizing framework spelled out above.

A conventional definition of MVP as applied by conservation biologists is: "A minimum viable population for any given species in any given habitat is the smallest isolated population having a 99% chance of remaining extant for 1,000 years despite the foreseeable effects of demographic, environmental and genetic stochasticity, and natural catastrophes" (Primack, 1998, p. 280). Of course, it is a matter of taste as to what probability (90% versus 99%?) and time horizon (100 versus 1000 years?) should be applied. The point is that the MVP concept tries to provide a quantitative estimate of how large a population should be to ensure long-term survival. Conservation biologists are quick to point out that reducing the species viability to a single magic number is likely too simple and cannot do justice to the interaction (feedback and synergy) of the different variables and processes involved. More complex, population viability analysis is necessary to assess the prospects of real populations facing actual challenges and risks, rather than considering stylized principles. In addition, there is the risk that MVP estimates are taken literally, rather than as cautionary guidelines.[7] In spite of these (rather well-

[7] For example, it may be argued that some species are too rare to be viable. Especially large predators (e.g., tigers, jaguars) display levels of abundance in certain regions that are known to be well below reasonable MVP estimates. While such small populations are clearly under threat, conservation biologists warn against using the MVP concept as a scapegoat to give up on these species entirely. As mentioned, MVP requirements are species-dependent (hence the lessons from bighorn sheep do not readily spill-over to tiger conservation), and in addition there are (many) examples of populations that have made a comeback after being depleted to extremely low levels (see footnote 1). A prerequisite for such recovery is obviously that habitat requirements are satisfied, which is questionable for many large mammal species.

known) objections, consider the available evidence.

Estimates of MVP obviously depend on the characteristics of the species concerned. While detailed demographic studies of the species and analyses of its environment are necessary to produce MVP estimates, there are some general insights. Evidence from studies of such diverse animals as birds and bighorn sheep, for example, clearly indicate that short- or medium-run survival (up to 50 or 100 years) is jeopardized for populations below 50–100 individuals. For the medium-run, theoretical models suggest that environmental variation is more important than demographic variation in determining survival probabilities and, hence, MVP requirements. This is not surprising, given that environmental stochasticity operates at the level of the population, whereas demographic stochasticity is first and foremost something that happens at the level of the individual. If we are interested in the long run, however, genetic considerations come into play and likely become the limiting factor.

For vertebrate species, biologists have considered how many animals should be protected to preserve genetic variability and allow the species to recover after being hit by a catastrophe (Soulé, 1987; Quammen, 1996; Primack, 1998). While an effective population of about 50 individuals is likely enough to avoid (short-term) inbreeding depression, no less than 500 (but possibly as many as 5000) individuals are required to balance (long-term) genetic drift.[8] The 50–500 rule, although often misused, may serve as a first rule of thumb in assessing species viability. Soulé (1987) remarks that MVPs for most species are likely in the "low thousands", but also stresses the uncertainty surrounding this number. If we are interested in the long-run survival of species, it is likely best to adopt the rather restrictive (but workable) assumption that policy should aim to maintain at least 2000 individuals of each species (e.g., grizzly bears, elephants), and perhaps more for some.

[8] The census population is defined as the total number of living animals in the population, whereas the effective population is a mathematically derived number reflecting patterns of breeding.

6. A fuzzy compromise

As a metaphor, consider that the economist is concerned only with economic efficiency, while the biologist desires only species numbers and their survival. Now, the economist's optimum stock of a biological resource may be greater or smaller than the biologist's minimum stock. Assume that, whenever $x^* > \text{MVP}$, there is no conflict as long as biologists always prefer larger stocks of a species to smaller ones. However, since there are often considerable opportunity costs involved in conservation of habitat and species, we can safely argue that for certain species the economic stock will be smaller than MVP (Bulte and van Kooten, 2000). The potential conflict between the economist's and biologist's benchmark is further illustrated by considering the effect of uncertainty on preferred stock size. If a population is subject to more intense stochastic perturbations, the biologist's response is to increase unambiguously the MVP estimate. For the economist, on the other hand, such perturbations may render the population a less attractive asset to invest in, and thus lead to downward revisions of the optimal stock.

In the case of conflict, which benchmark should prevail? Since human resources are scarce, extinction of some species is almost inevitable (Mann and Plummer, 1995). To what extent should our biologist compromise her ethical principles? Realizing that species conservation may come at the cost of other worthy objectives (health care, poverty alleviation), it is not clear that a position of protecting all species (or species at all cost) holds the moral high ground (Shogren, 1998). And to what extent should our economist be willing to sacrifice social income (efficiency) to protect species? Clearly, answering such questions is beyond the realm of standard economic science; therefore, we propose one method that can be used to structure thinking about such awkward tradeoffs.

In what follows, we aim to balance potentially conflicting desires with respect to renewable resource management. Because of the ethical nature of the issues involved, it is by no means obvious that efficiency considerations should dominate the decision. As Sagoff (1988) notes: "... it is not just a matter of balancing interests with interests, it is a matter of balancing interests with morality and balancing one morality with another morality'' (p. 98). In other words, while economics is important in designing instruments for achieving certain objectives (e.g., so that they are reached in the most cost-effective manner), the same reasoning should perhaps not be allowed to determine the goals themselves (see Common, 1995; Toman, 1994).

Balancing ethical and economic interests implies making tradeoffs that cause one to deviate from benchmarks, such as the economist's optimal population. What is required is that the objectives of economic science and those of conservation biology are each satisfied to a lesser degree than if decisions are reached in disciplinary isolation. Inherent in such a tradeoff is thinking about uncertainty that is different than that of Gambler's Ruin type of stochasticity. This form of uncertainty is more appropriately referred to as vagueness (Dubois and Prade, 1993), and it cannot typically be resolved using probability theory (Barret and Pattanik, 1989; Kosko, 1992; Dubois and Prade, 1993). Even when all information about the resource (MVP, growth rates, opportunity cost, etc.) is fully known, there remains uncertainty about how one discipline's view of what is ''best'' should be weighed relative to that of another. Non-probabilistic, multiple-objective programming (NPMOP) is one means that can be used to incorporate uncertainty of this type (Zimmermann, 1996; Ells et al., 1997; Krcmar et al., 2000).

It is beyond the scope of this paper to treat non-probabilistic methods in detail, but we provide some basic notions as these relate to the fuzzy logic that underpins NPMOP. The key issue in fuzzy modeling is that elements can have different degrees of membership in fuzzy sets. In ordinary calculus, an element z of the universal set Z is assigned to a set A via the characteristic function μ_A such that:

$$\begin{pmatrix} \mu_A(z)=1 & \text{if } z\in A\subset Z, \\ \mu_A(z)=0 & \text{otherwise} \end{pmatrix} \tag{9}$$

Hence, the element has either full membership ($\mu_A(z)=1$) or no membership ($\mu_A(z)=0$) in the ordinary (or crisp) set A. Crisp valuation is binary, or $\{0, 1\}$. Ordinary sets are useful for many classifications (e.g., a person responding to a survey is either male or female), but fuzzy sets, in contrast, are described by a characteristic function that maps over the closed interval [0, 1]. More formally, a fuzzy set $\tilde{A}$ of the universal set Z is defined by the membership function $\mu_{\tilde{A}}$: $Z\rightarrow[0, 1]$, which assigns to each element $z\in\tilde{A}$ a real number $\mu_{\tilde{A}}(z)$ in the interval [0, 1], where the value of $\mu_{\tilde{A}}$ at z represents the degree of membership of z in the fuzzy set $\tilde{A}$.

Fuzzy sets are useful when the boundaries of a set are ill defined. For example, consider the set ''natural forests''. Some ecosystems (such as those on Canada's Pacific coast) qualify as members of this set with degree of membership equal to one because they have never experienced human development (they have never been logged). However, as the level of human intrusion (and number of trees affected by human actions) increases, the 'natural' classification *gradually* becomes less apt. The degree of membership declines and ultimately falls to zero for forest ecosystems that have been clear felled. Similar reasoning applies to the fuzzy sets 'sustainable population' and 'economically optimal population'.

Denote by $\tilde{O}$ the fuzzy set associated with the 'economically optimal population' (determined in Section 4), and by $\tilde{S}$ the 'sustainable population'. The economically optimal stock x^* (from Eq. (8)) would have degree of membership equal to one in the set $\tilde{O}$: $\mu_{\tilde{O}}(x^*)=1$. As actual stock at time t, $x(t)$, deviates from the benchmark x^*, the degree of membership falls. The fuzzy set $\tilde{O}$, 'optimal population', is a two-sided fuzzy set as both greater and lower abundance than x^* reduce the degree of membership. The ''sustainable population'', on the other hand, can be represented by a one-sided fuzzy set, since $\mu_{\tilde{S}}(x^*)$ increases as $x(t)$ increases above MVP — increasing the size of the in situ stock does not harm sustainability. Note here that we assume that both MVP and x^* are crisp.

It is the task of the researcher to construct the appropriate fuzzy sets. While membership functions can take on a variety of forms, linear specifications are often employed (Zimmermann 1996;

Kosko 1992). In NPMOP, we are concerned with uncertainty about definitions of the sustainable and optimal populations for a certain species, or fuzzy sets $\tilde{S}$ and $\tilde{O}$, respectively. The problem can be illustrated with the aid of Fig. 1.

In Fig. 1, the membership of the fuzzy set 'sustainable population' $\tilde{S}$ can be defined as:

$$\begin{aligned} &\mu_{\tilde{S}}(x) = 0 && \text{if } x < \text{MVP} - \alpha \\ &\mu_{\tilde{S}}(x) = 1 - \frac{\text{MPV} - x}{\alpha} && \text{if } \text{MVP} - \alpha \leq x < \text{MVP} \\ &\mu_{\tilde{S}}(x) = 1 && \text{if } x \geq \text{MVP} \end{aligned} \tag{10}$$

where α is the left-side 'spread' (not to be confused with APLHA in section 4). In equation Eq. (10), we assume that maintaining a stock greater than or equal to MVP ensures species survival (full membership in the fuzzy set 'sustainable populations') and that further additions to the stock are unimportant for its future survivability. For stocks smaller than MVP, viability is compromised and survival is at risk (and the degree of risk depends on the deviation from the MVP benchmark).[9] Populations whose abundance is depressed below the boundary defined by MVP $-\ \alpha$ do not stand a chance and will go extinct in the foreseeable future. Consistent with the discussion in preceding sections, the degree of membership increases monotonically as the species becomes more abundant. The spread is to be determined by the researcher and should be based on biological arguments, preferably in an interactive fashion with biologists. While the membership function for $\tilde{S}$ is linear and one-sided, other specifications (including non-linear ones) are possible.

Membership in the 'optimal population' fuzzy set $\tilde{O}$ can likewise be defined. In Fig. 1, the following linear functional form is employed:

$$\begin{aligned} &\mu_{\tilde{O}}(x) = 0 && \text{if } x \leq x^* - \phi \\ &\mu_{\tilde{O}} = 1 + \frac{x - x^*}{\phi} && \text{if } x^* - \phi < x < x^* \\ &\mu_{\tilde{O}}(x) = 1 && \text{if } x = x^* \\ &\mu_{\tilde{O}}(x) = 1 + \frac{x^* - x}{\phi} && \text{if } x^* < x < x^* + \phi \\ &\mu_{\tilde{O}}(x) = 0 && \text{if } x \geq x^* + \phi \end{aligned} \tag{11}$$

where ϕ is the spread, measuring the interval over which the renewable resource yields satisfactory outcomes in terms of the economic criterion. Based on the economic performance of the various population levels and the opportunity costs of conservation, economic considerations (and economic experts) should dictate this function. Fuzzy membership function Eq. (11) states that population levels below $x^* - \phi$ and above $x^* + \phi$ yield no appreciable economic benefits, perhaps because harvesting costs are excessive in the former case or the species in question becomes a nuisance in the latter.

The decision space is defined by the intersection of the fuzzy objectives (the intersection $\tilde{O} \cap \tilde{S}$ is the smallest fuzzy set contained in both $\tilde{O}$ and $\tilde{S}$):

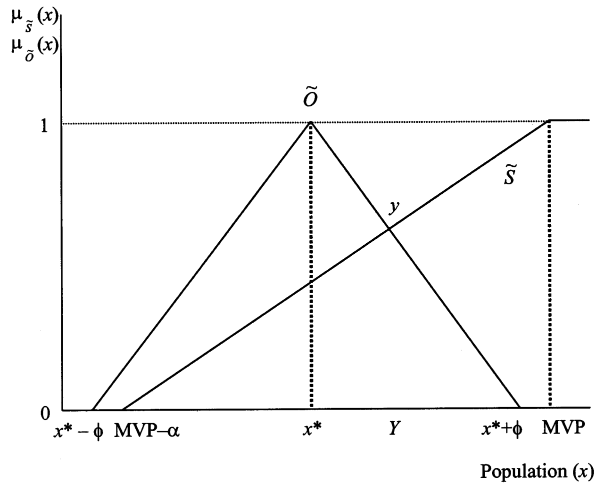

Fig. 1. Membership functions for 'optimal population' and 'sustainable population'.

[9] While MVP is assumed to be a crisp value, there is some ability to save a species should population numbers fall below MVP. The size of the spread depends on the possibility of using science to bring back to life a species that has slipped below its minimum viable population; given that this is possible, in some cases, but not desirable, implies that $\alpha > 0$. Alternatively, one can think of MVP as being fuzzy, in which case one fuzzy number (minimum viable population) gets imbedded in another (sustainable population) (see Bandemer and Gottwald, 1996). None of this changes the ideas developed in the text, however.

$\mu_{\tilde{O}\cap\tilde{S}}(x)=\min[\mu_{\tilde{O}}(x),\mu_{\tilde{S}}(x)]$. In the example of Fig. 1, this is given by the triangle MVP$-\alpha$, y, $x^*+\phi$.

One possible approach to balance conflicting objectives is to maximize the minimum degree of satisfaction (membership) of both objectives, such that the compromise solution is obtained when the lowest level of satisfaction has been raised as high as possible (but see Mendoza and Sprouse, 1989). The maxmin operator is:

$$\max\mu_{\tilde{O}\cap\tilde{S}}(x)=\max\ \min[\mu_{\tilde{O}}(x),\mu_{\tilde{S}}(x)]. \tag{12}$$

Assuming linear membership functions, Zimmermann (1996) demonstrates that a NPMOP with vague and conflicting objectives can be formulated as an ordinary linear programming (LP) program as:

$$\begin{aligned} &\max \quad \lambda \\ \text{subject to} \quad & x-b_i-d_i(\lambda-1)\geq 0 \quad i=1,2,3 \\ & \lambda\in[0,1] \\ & x>0 \end{aligned} \tag{13}$$

where $\lambda=\mu_{\tilde{O}\cap\tilde{S}}(x)$ b_i is the threshold value for the constraint under consideration, and d_i is the spread of the corresponding one-sided fuzzy set (β or α).[10] The stock size that maximizes λ (or Y in Fig. 1) is the socially preferred population level as it is Pareto efficient in the sense that any deviation from that population level will make either the biologist or economist worse off. Specifically, a move to the "right" from the equilibrium Y in Fig. 1 will make the economist worse off (that is, the degree of membership in fuzzy set $\tilde{O}$ falls) whereas a move to the left will make the ecologist worse off. The compromise population may be considered fair because (1) both our economist and biologist had input into the construction of their preferences pertaining to the population levels that satisfy their concern for economic efficiency and biological ethics, respectively. In addition, (2) the stock size maximizing guarantees that economists and ecologists are *equally* satisfied (as represented by the degree of membership in their respective fuzzy sets of acceptable outcomes). Previous research using fuzzy multiple objective goal programming indicated that the recommendations of such models may be quite consistent with the outcomes of a negotiating process where various stakeholders aim to achieve a consensus (e.g., Ells et al., 1997). This suggests that the compromise outcomes may be more politically acceptable than management recommendations based on a single criterion (be it welfare maximization or species survival).[11]

Refer to Fig. 1 again, and imagine that ecologists and economists offer conflicting advice on the management of a salmon stock on Canada's west coast (a motivating example in the introduction). Economists prescribe steady state harvesting associated with stock size x^*, while ecologists fear that such management may jeopardize the stock's survival. They propose a steady state stock of size MVP, resulting in a significant loss of the present value of welfare. What should managers decide? Given the spreads as defined in Eqs. (10) and (11), our model offers a compromise solution. To balance concerns about long-run viability and welfare, the stock size Y is proposed. While both economists and ecologists may be somewhat disappointed about this result, neither discipline can be appeased without hurting the other. This compromise stock is a function of the benchmarks and the spreads (steeper spreads for one benchmark are consistent with a compromise solution that is closer to that benchmark), and computing it is thus conditional on the willingness and ability of scientists from different disciplines to provide honest input.

[10] The two-sided fuzzy set can be rewritten as two one-sided sets, but this only increases the number of constraints (by the number of two-sided fuzzy variables).

[11] Toman (1994) and Farmer and Randall (1998) discuss a two-tiered decision making process in which, depending on the context, resource management is guided by standard economic efficiency considerations or by the safe minimum standards concept (see text). The compromise model may be consistent with such reasoning. When stocks are greater than the MVP benchmark as defined by ecologists (and, hence, membership in the fuzzy set sustainable populations is equal to unity) management may be driven by maximization of welfare. If stocks fall below the MVP value, management should be guided by the maximization of λ instead.

7. Discussion

Conventional bioeconomic models, including those that explicitly incorporate minimum viable populations, are not consistent with the insights of conservation biology. Here we demonstrated how optimal management policies are affected by a more realistic (and complex) biological underpinning that includes demographic and environmental variability, as well as catastrophes. Consistent with prior work by Pindyck (1984), Olsen and Shortle (1996) and Bulte and van Kooten (1999b), the net effect of stochasticity, including the implications of Jensen's inequality, on optimal stock size is ambiguous.

We also discussed that the optimal stock thus defined may conflict with the MVP concept as used by conservation biologists, and offered a fuzzy compromise between the economic and ecological benchmarks that can be applied when the first falls short of the latter. In applied environmental economics, ethical issues are often involved, and it is not at all obvious that efficiency criteria should prevail. The safe minimum standard (SMS) represents one attempt to address critical thresholds that do not fit neatly under the efficiency umbrella; when dealing with potential irreversibility, one decision criterion (economic efficiency) is jettisoned for another (SMS), but only if the costs of avoiding irreversibility are tolerable (van Kooten and Bulte 2000, pp. 247–249). Irreversibility appears to have a value that is not adequately captured by the efficiency criterion (cost–benefit analysis). As Farmer and Randall (1998) argue, because there is no consistent moral theory delineating duties of humans to each other and to future generations, it makes sense to abandon economic efficiency as a decision criterion and substitute another, even though the former remains valid in all other cases (see also Berrens et al., 1998). The fuzzy compromise presented in this paper seeks to balance more explicitly the economic efficiency criterion (even one where stochasticity is accounted for) with an ethical criterion rooted in conservation biology. It is more explicit because our economist and our biologist must construct the membership functions that represent their respective moral positions. Depending on the specification of the fuzzy set "sustainable populations" as defined by ecologists, the fuzzy compromise may be more or less consistent with management recommendations as following from the SMS concept.

Finally, consider again the question of population viability. We ignored the genetic dimension of conservation of small populations in our economic model, but would like to stress that including such considerations may severely complicate the analysis. Genetic drift and inbreeding suggest that the *lowest* historical population may be relevant for population viability, rather than the current stock, thus violating one of the basic properties of the Markov processes. This lowest level then acts as a bottleneck through which the population has moved in terms of genetic variation, reducing the variability in alleles present and restricting the ability to respond to future changes in the natural environment and reducing fitness.[12] Also, inbreeding may exacerbate demographic stochasticity (Lande and Barrowclough, 1987), which indicates that the various factors affecting population viability are not simply additive, further complicating formal analyses. Moreover, due to ongoing genetic drift and inbreeding of small populations (or populations that have once been reduced to low levels), the entire steady state concept may be open to dispute.

Economic issues related to the minimum viable population need to be resolved as well. For example, non-use values are modeled as a function of species abundance (as in this study), while they are likely more dependent on whether a species will survive or not (i.e., on MVP). While this only strengthens the case for modeling sustainable populations as a fuzzy set, it also raises the question

[12] Genetic stochasticity therefore has an impact on the economically optimal steady state stock x^* (the species in question represents a less favorable investment after it has passed through a bottleneck in the past), and on the "steady state" harvest level. But the problem is actually more complex than this, because *approach dynamics* may also be affected. For example, in the context of models with irreversible investments in harvest gear Clark et al. (1979) Boyce (1995) demonstrate that it may be optimal to depress temporarily in situ populations to low levels. It remains to be seen whether it is actually optimal to reduce stocks to levels smaller than the steady state when such a policy implies that the reproductive potential of the species is affected.

about whether non-use values themselves are fuzzy. We can only argue that these and other issues need to be left for future research (van Kooten et al, 2001).

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