# A Numerical Investigation of Izhikevich's Simple Model of Spiking Neurons

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## 1 Introduction

Neuroscience is a particularly difficult field to study, not only due to the complexity of the human nervous system, but also because collection of empirical data is incredibly difficult. The nervous system, especially the brain, is extremely delicate, raising technical and ethical issues in its physiological study. This is why neuroscience is a field in which physical experimentation is best complemented by theoretical modeling and computational simulations. The first biologically accurate mathematical model of neuron spiking is attributed to a series of papers published by Alan Hodgkin and Andrew Huxley in 1952 (Hodgkin & Huxley, 1952). Since its publication, and the subsequent integration of computer science into the field, researchers have worked to create models that are computationally efficient, while still remaining as biologically plausible as the Hodgkin-Huxley model. In 2003, Eugene M. Izhikevich published his paper, a "Simple Model of Spiking Neurons", outlining the model he developed to address the previously stated needs of the field (Izhikevich, 2003). The model's derivation, based on bifurcation methodologies, normal form reduction, and the quadratic integrate-and-fire model, is described in Izhikevich's monolith (Izhikevich, 2006).

In this project, I implement Izhikevich's model utilizing Maple software to reproduce characteristic spiking patterns of cortical neurons. In addition to this, I also adapt one of his simulations into the Python programming language, in order to study how sensitive cortical neuron networks are to respective ratios of inhibitory and excitatory neurons.

# 2 Background

## 2.1 The Cerebral Cortex

Cortical neurons are located within the Cerebral Cortex, an outer layer of grey matter tissue situated directly above the Cerebrum (Javed et al., 2023). This area of the brain is responsible for the integration of sensory information and inputs, which are then used to make decisions that affect a human's behavior, personality, memory, and complex thinking processes. The neurons that make up the Cerebral Cortex synapse with one another to form complicated and extended neural networks. Within these networks, the cortical neurons will exchange information and associate to perform specific functions.

The Cerebral Cortex is one of the best understood areas of the brain. Not only is it essential to the everyday functions of human beings, but it's location on the outer surface of the brain makes collecting data relatively easy.

## 2.2 Neuron Spiking

The way neurons actually communicate with one another is through the generation and propagation of action potentials. Action potentials are rapid changes in voltage across the neuronal membrane that, by reaching a certain threshold voltage, are able to transfer information along their own cell bodies and to neighboring neurons (ie. neurons they synapse with). The below graphic is a useful way to visualize an action potential, and also includes the biochemical features of each phase.

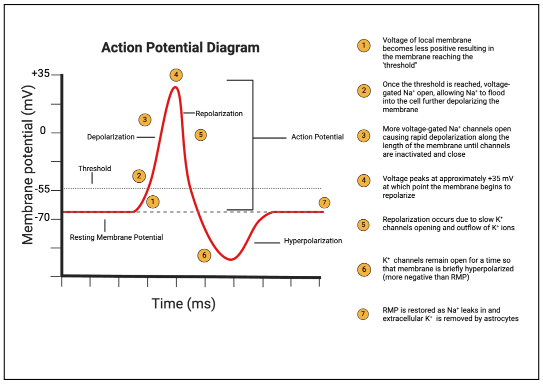


Figure 1: Action potential diagram

### 2.3 Izhikevich's Model

The system of ordinary differential equations that makes up Izhikevich's model is stated below:

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I \tag{1}$$

$$\frac{du}{dt} = a(bv - u) \tag{2}$$

with the following post-spike behaviors:

$$if\ v \geq 30mV,\ then \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \tag{3}$$

#### 2.3.1 Variables and Parameters

| | Name | Description |
|---|---|---|
| $v$ | Membrane Potential | The difference in voltage across the neuronal membrane |
| $u$ | Membrane Recovery | Activation of $K^+$ and inactivation of $Na^+$ ion currents |
| $a$ | Time Scale of Recovery Variable | Scales $u$ - smaller values correlate with slower after-spike recovery |
| $b$ | Membrane Potential and Recovery Coupling | Sensitivity of $u$ to sub-threshold activations of $v$ - larger values correlate with stronger couplings |
| $c$ | After-Spike Reset ($v$) | Caused by fast, high-threshold $K^+$ conductances |
| $d$ | After-Spike Reset ($u$) | Caused by slow, high-threshold $Na^+$ and $K^+$ conductances |
| $I$ | Input Current | Synaptic or injected current |

## 3 Methods

### 3.1 Maple Program

The full implementation of the model, using Maple 2025 software, can be found here: https://sites.math.rutgers.edu/ zeilberg/Bio25/Projects/Proj6.txt. The values of parameters $a$, $b$, $c$, and $d$ were adjusted to produce different neuron spiking patterns, which were then visualized in plots.

### 3.2 Python Program

The full implementation of the simulation, using Python 3.15 and the integrated development environment (IDE) Spyder 6, can be found here: https://sites.math.rutgers.edu/ zeilberg/Bio25/Projects/Proj6.txt. Within the simulation, the ratio of inhibitory to excitatory cortical neurons was adjusted to produce different network behaviors and wave distributions.

# 4 Results

## 4.1 Neuron Spiking Patterns

| **Spiking Pattern** | **a** | **b** | **c** | **d** |
|---|---|---|---|---|
| Regular Spiking (RS) | 0.02 | 0.2 | -65 | 8 |
| Intrinsically Bursting (IB) | 0.02 | 0.2 | -55 | 4 |
| Chattering (CH) | 0.02 | 0.2 | -50 | 2 |
| Fast Spiking (FS) | 0.1 | 0.2 | -65 | 2 |
| Thalamo-Cortical (TC) | 0.02 | 0.25 | -65 | 0.05 |
| Resonator (RZ) | 0.1 | 0.26 | -65 | 8 |
| Low-Threshold Spiking (LTS) | 0.02 | 0.25 | -65 | 2 |

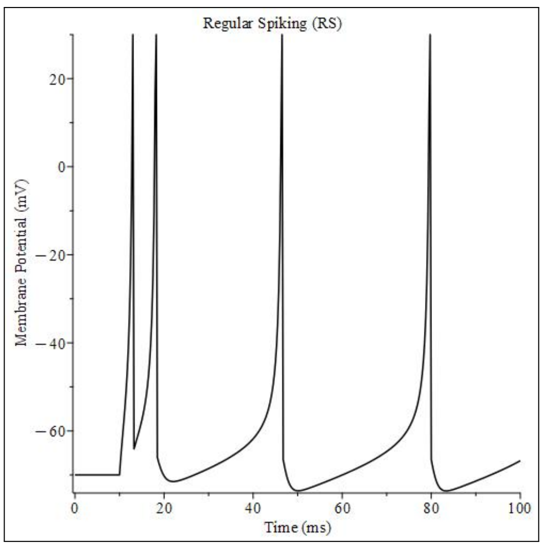


Figure 2: RS, $a = 0.02$, $b = 0.2$, $c = -65$, $d = 8$

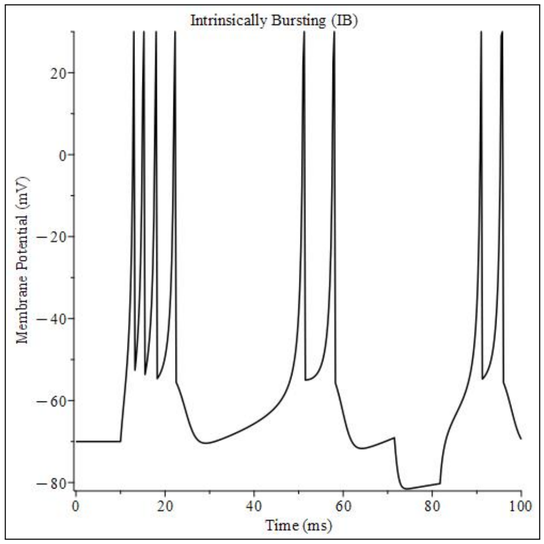


Figure 3: IB, $a = 0.02$, $b = 0.2$, $c = -55$, $d = 4$

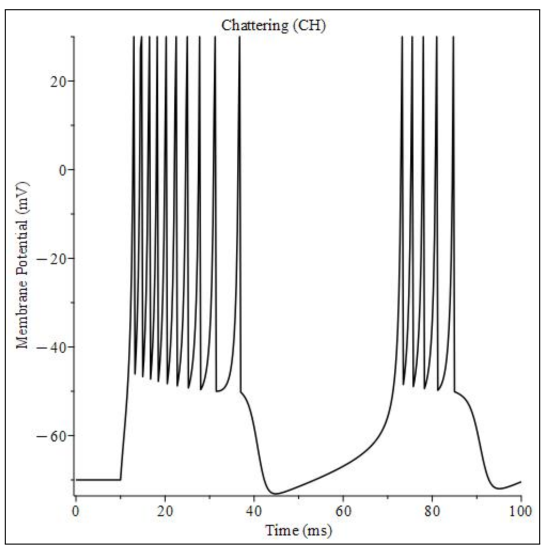


Figure 4: CH, $a = 0.02$, $b = 0.2$, $c = -50$, $d = 2$

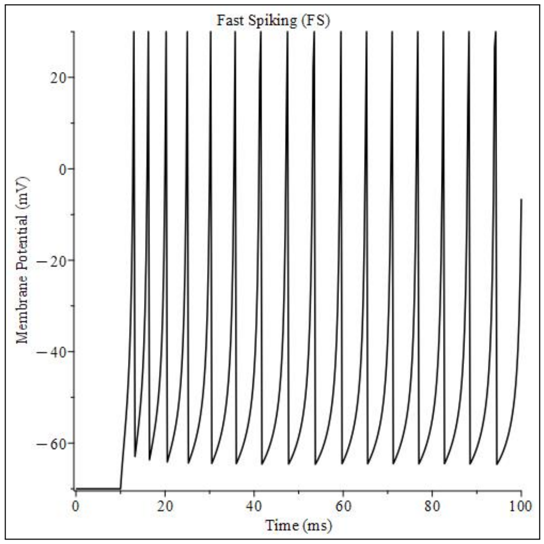


Figure 5: FS, $a = 0.1$, $b = 0.2$, $c = -65$, $d = 2$

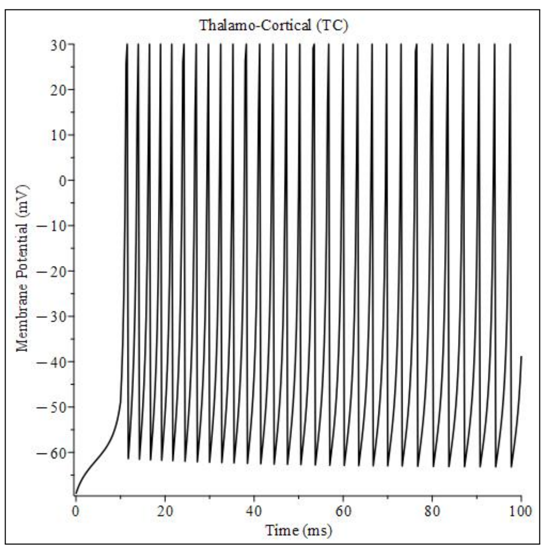


Figure 6: TC, $a = 0.02$, $b = 0.25$, $c = -65$, $d = 0.05$

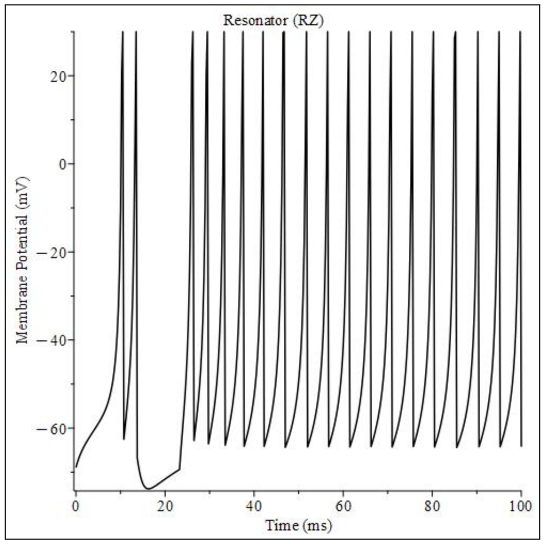


Figure 7: RZ, $a = 0.1$, $b = 0.26$, $c = -65$, $d = 8$

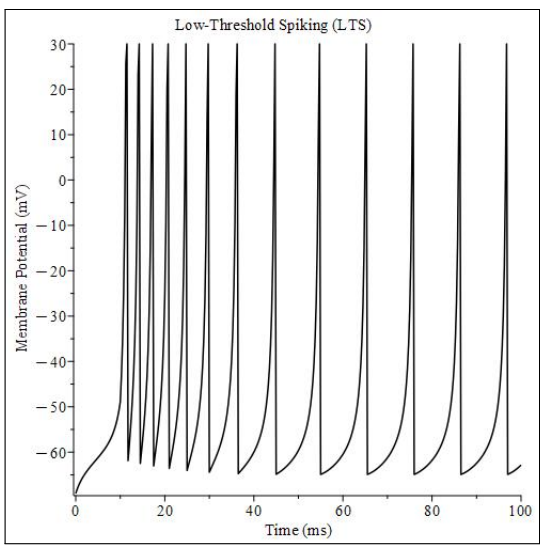


Figure 8: LTS, $a = 0.02$, $b = 0.25$, $c = -65$, $d = 2$

## 4.2 Neuron Network Simulation

Where "Ni" represents the number of inhibitory neurons and "Ne" represents the number of excitatory neurons in a network containing a total of 1,000 neurons.

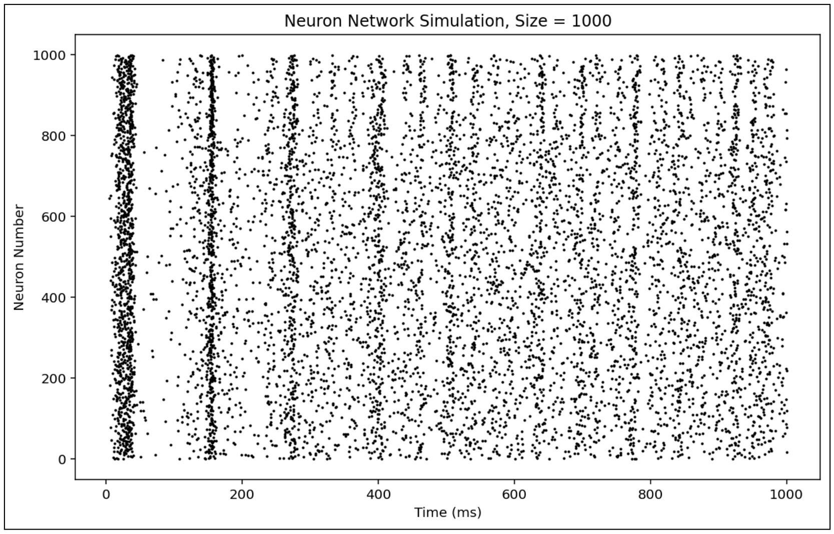


Figure 9: Typical mammalian inhibitory/excitatory neuron ratio, $(Ni = 200, Ne = 800)$

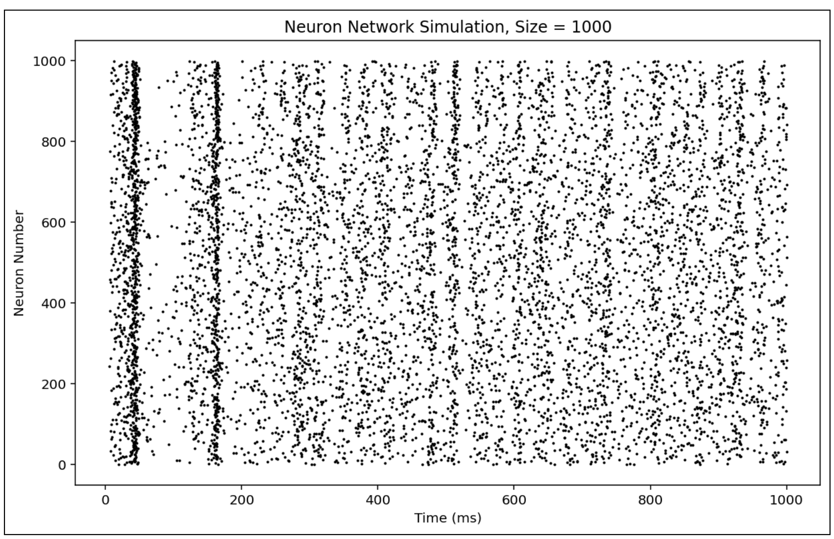


Figure 10: Moderately abnormal mammalian inhibitory/excitatory neuron ratio, $(Ni = 195, Ne = 805)$

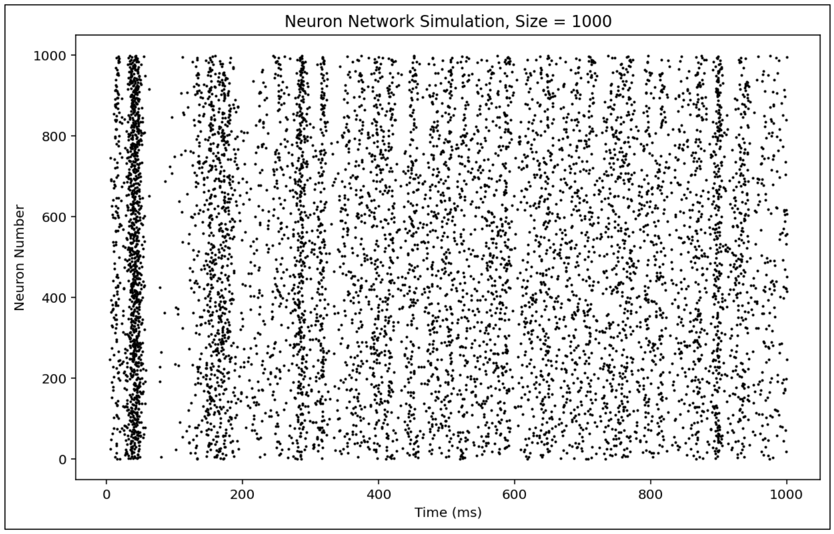


Figure 11: Severely abnormal mammalian inhibitory/excitatory neuron ratio, $(Ni = 190, Ne = 810)$

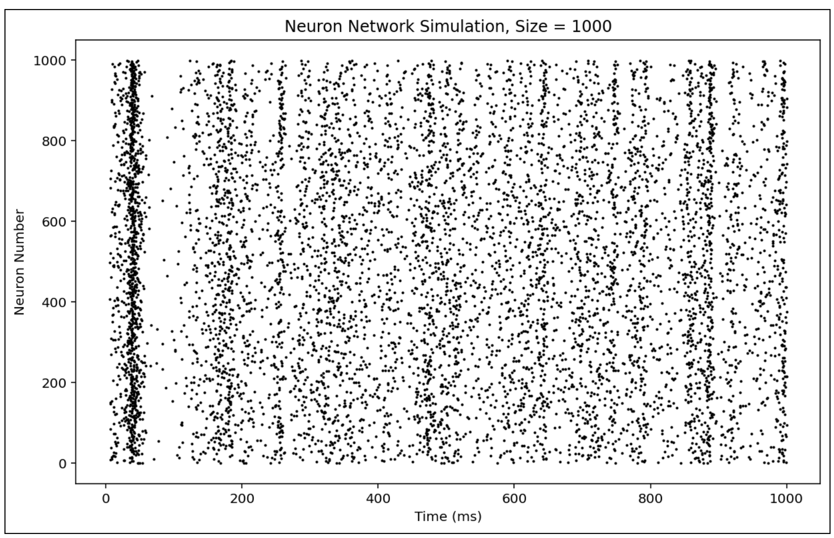


Figure 12: Moderately abnormal mammalian inhibitory/excitatory neuron ratio, $(Ni = 205, Ne = 795)$

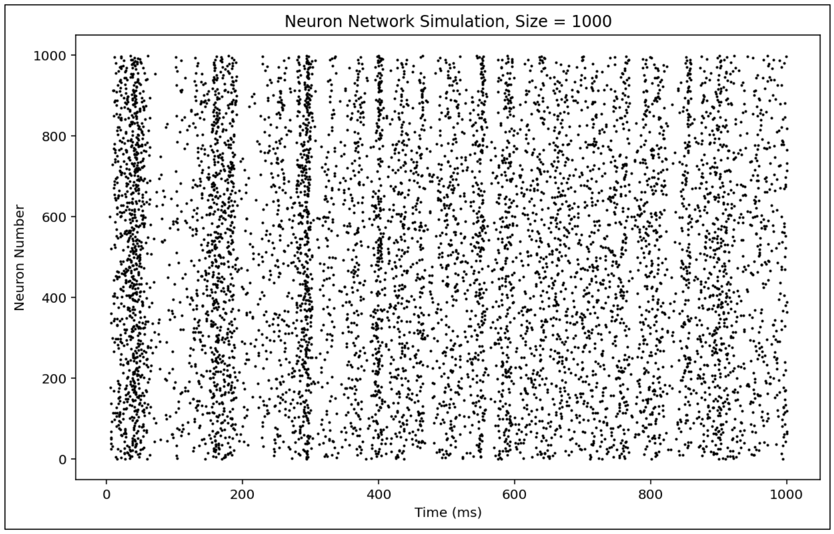


Figure 13: Severely abnormal mammalian inhibitory/excitatory neuron ratio, ($Ni = 210, Ne = 790$)

# 5 Discussion

## 5.1 Neuron Spiking Patterns

The Regular Spiking (RS), Intrinsically Bursting (IB), and Chattering (CH) neurons are all sub-types of excitatory cortical neurons. For RS neurons, their initial response to a stimulus is a series of small spikes. Eventually, there is a frequency adaptation and the inter-spike period increases with increasing time. This is caused by the hyper-polarization that occurs after each spike, causing absolute and relative refractory periods. These periods represent intervals of time in which a single neuron must pause before releasing another action potential. IB neurons will first respond to an input with a bursting pattern of closely timed spikes. This is able to occur because after the end phase of one spike, this particular type of neuron does not return to it's resting membrane potential, nor does it hyper-polarize beyond it. After this first burst, IB neurons will experience adaptation and begin to exhibit a spiking pattern closer to that of a RS neuron. CH neurons send out multiple bursts, with fairly consistent refractory periods between each event.

The Fast Spiking (FS) and Low-Threshold Spiking (LTS) neurons are both sub-classes of inhibitory cortical neurons. FS neurons fire singular action potentials at a high frequency because their fast recovery parameter prevents the adaptation or slowing down of its spiking pattern. LTS neurons are similar to FS neurons in that they fire action potentials at high frequency, however, this kind of neuron will eventually experience adaptation.

Izhikevich's model can accurately reflect the behavior of Thalamo-Cortical (TC) neurons, whose axons extend between the Cerebral Cortex and the Thalamus, an area of the brain that acts as a gateway for signal transmission. Though TC neurons can exhibit multiple kinds of spiking patterns, when they start at a resting membrane potential, they exhibit a firing behavior similar to that of RS neurons

This model is also able to produce Resonator (RZ) dynamics, in which a neuron can respond to subthreshold oscillations. These resonate to the rhythm of the inputs, rather than their magnitudes.

## 5.2 Neuron Network Simulation

In addition to the modeling of individual neurons, Izhikevich's model can also accurately reproduce the behavior of neuron networks. To reiterate, a neuron network is a group of neurons that synapse and associate with one another to collectively perform a specific function or task. One important feature of the plots derived from the neuron network simulations is the difference between its dense and sparse regions. At the denser regions, where a distinct vertical line can be seen, a majority of the neurons will fire at the same time. Within the sparser regions, individual neurons do not tend to collectively fire, leading to a very random looking output pattern.

Notice that denser areas do not occur frequently, or for much longer after the initial stimulation is given to the network. These regions actually correspond to alpha brain waves, which range in frequency from 8-12 Hz. These wave types are usually present when an individual is performing a task that requires a moderate amount of effort or attention. On the other hand, the sparse regions where neurons activate more often, are representative of gamma brain waves, ranging from 30-90 Hz. These occur when an individual performs cognitive and motor tasks that require a large deal of energy and focus. The type of brain waves a neuron network exhibits can tell a great deal about the function and behavior of that particular network.

## 6 Conclusion

The analysis of both single and collective neuron spiking behavior, made computationally possible by Izhikevich's model, can reveal a lot of valuable information about the functions of a neuron or network. Changes in spiking dynamics of individual neurons is thought to relate to the ever-shifting topology of neuron networks, and even entire brain areas (Yang et al., 2025). From the change in behavior of just one neuron, entire neural processes can be elucidated. The neuron network simulation is helpful in that it reveals how an adjustment in the ratio of inhibitory to excitatory neurons is displayed in the of size and distribution of alpha and gamma brain waves. Interestingly, disruption to brain wave patterns is often indicated in many neuro-pathologies, such as Alzheimer's Disease, Depression, and Tinnitus (Guan et al., 2022). Trial therapies for these and other disease of the central nervous system often include using electrical stimulation to increase the frequency of gamma waves.

## 7 References

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