A NUMERICAL INVESTIGATION OF IZHIKENICH'S SIMPLE MODEL OF SPIKING NEURONS

MATH 336 - Fall 2025 - Final Project

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INTRODUCTION

- The Study of Neuroscience
- From Hodgkin-Huxley to Izhikevich
- Project Goals

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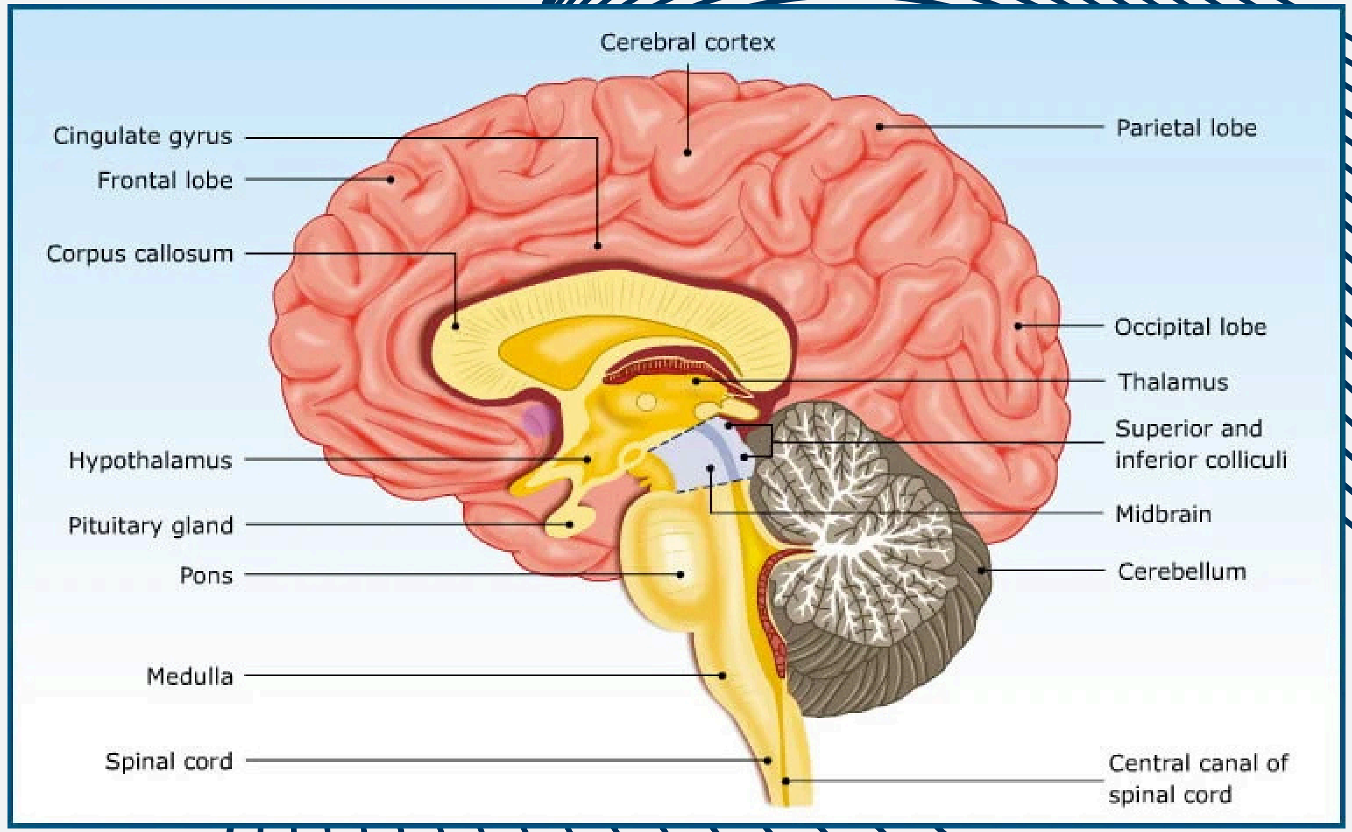
BACKGROUND - THE CEREBRAL CORTEX

- Location: Above Cerebrum
- Structure: Layers of grey matter (neuronal cell bodies) tissue
- Functions: Information associate and integration affecting behavior, personality, memory, high-order processes



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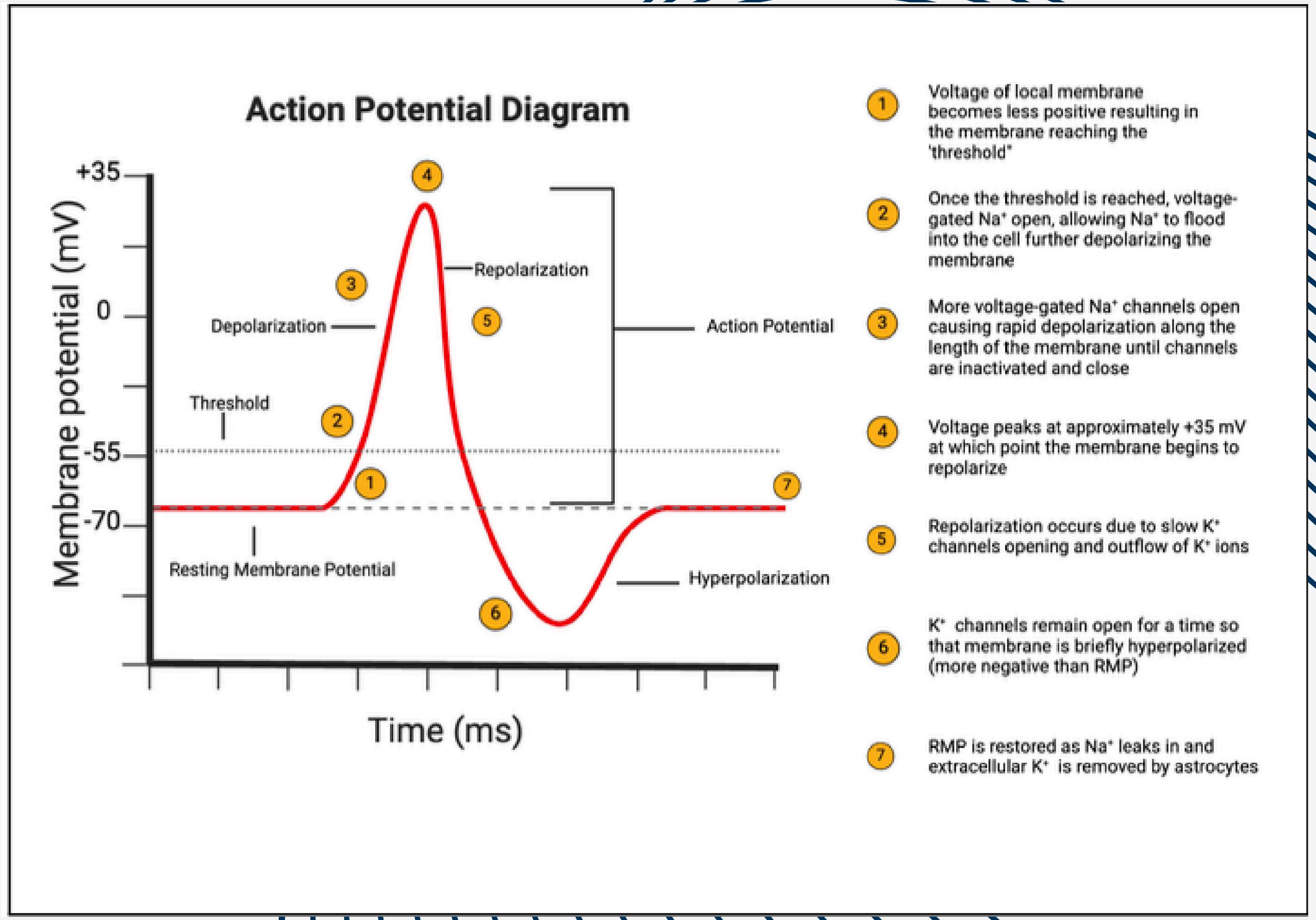
BACKGROUND - NEURON SPIKING

- How do neurons communicate with one another?
- What is the mechanism by which they do so?
- How do neurons associate to perform complex tasks?



BACKGROUND - IZHIKEVICH'S MODEL

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I$$

$$\frac{du}{dt} = a(bv - u)$$

$$if\ v \geq 30mV,\ then \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$$

	Name	Description
v	Membrane Potential	The difference in voltage across the neuronal membrane
u	Membrane Recovery	Activation of K^+ and inactivation of Na^+ ion currents
a	Time Scale of Recovery Variable	Scales u - smaller values correlate with slower after-spike recovery
b	Membrane Potential and Recovery Coupling	Sensitivity of u to sub-threshold activations of v - larger values correlate with stronger couplings
c	After-Spike Reset (v)	Caused by fast, high-threshold K^+ conductances
d	After-Spike Reset (u)	Caused by slow, high-threshold Na^+ and K^+ conductances
I	Input Current	Synaptic or injected current

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METHODS - MAPLE PROGRAM

```
SMNS := [0.04*v^2 + 5*v + 140 - u + i, a*(b*v - u)]

convert(EquP(SMNS, [u, v]), radical)
`Equilibrium points given the parameter values typical for mammalian cotrical neurons - observe that the equilibria are
complex, meaning the system has oscillatory behavior`
EquP(subs([a = 0.02, b = 0.2, c = -65, d = 2, i = 10], SMNS), [u, v])
```

```
`Plotting different neuron spiking dynamics by changing parameters`
`Figure 1: Regular Spiking (RS)`
a := 0.02;
b := 0.2;
c := -65.0;
d := 8.0;
v := -70.0;
u := b*v;
vv := [];
uu := [];
ts := 0.25;
N := round(100/ts);
tspan := [seq(evalf(q*ts), q = 0 .. N)];
T1 := tspan[-1]/10;
for q to N + 1 do
    t := tspan[q];
    if T1 < t then
        i := 14.0;
    else
        i := 0.;
    end if;
    v(x) := v(x) + 0.25*(0.04*diff(v(x), [x $ ~2]) + 5.0*v(x) + 140.0 - u + i);
    u := u + 0.25*a*(b*v(x) - u);
    if 30 < v(x) then
        vv := [op(vv), 30.0];
        v(x) := c;
        u := u + d;
    else
        vv := [op(vv), v(x)];
    end if;
    uu := [op(uu), u];
end do
n := nops(tspan);
P_1 := plots:-listplot([seq([tspan[q], vv[q]], q = 1 .. n)], color = black, style = line, title = "Regular Spiking (RS)",
labels = ["Time (ms)", "Membrane Potential (mV)"], labeldirections = ['horizontal', 'vertical']);
plots:-display(P_1);
```

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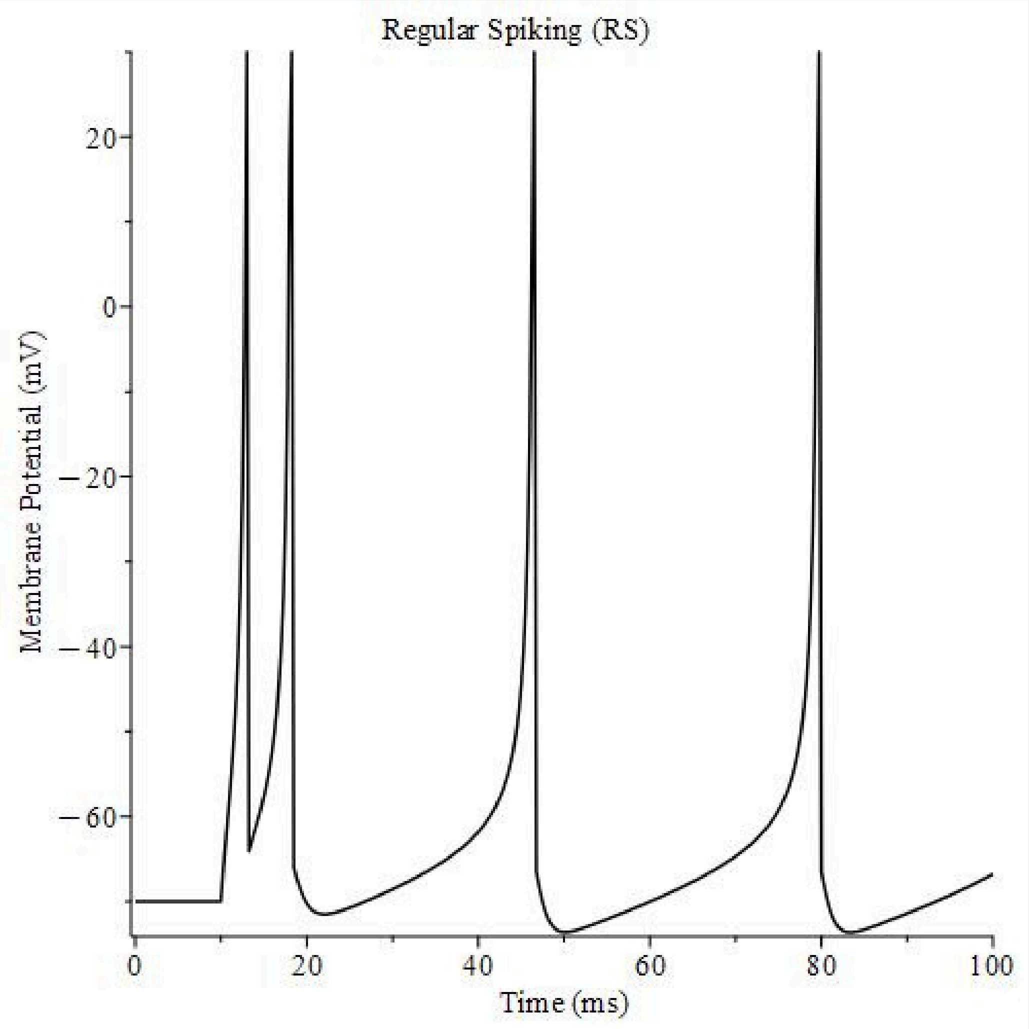
METHODS - PYTHON PROGRAM

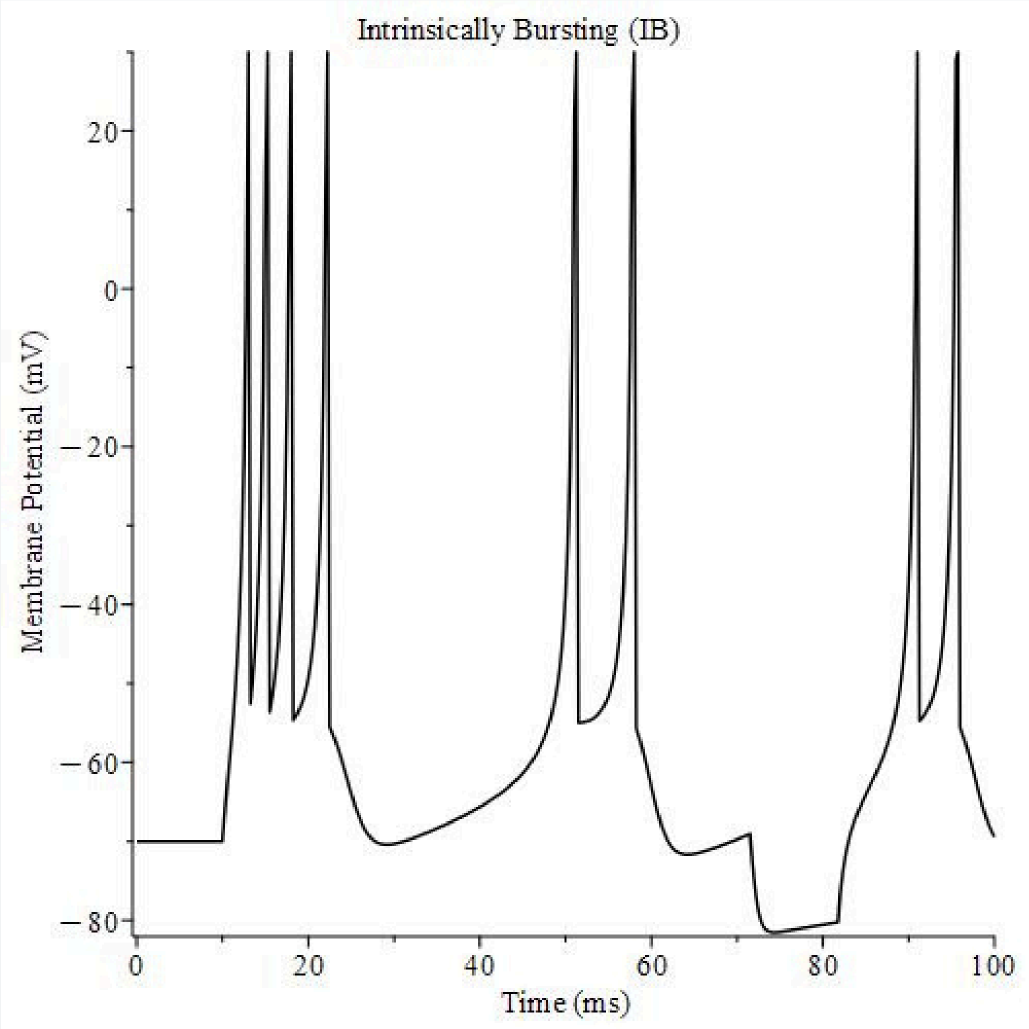
```
import numpy as np
import matplotlib.pyplot as plt

Ne = 800
Ni = 200
re = np.random.rand(Ne, 1)
ri = np.random.rand(Ni, 1)
a = np.vstack((0.02*np.ones((Ne,1)),0.02+0.08*ri)).flatten()
b = np.vstack((0.2*np.ones((Ne,1)),0.25-0.05*ri)).flatten()
c = np.vstack((-65+15*re**2,-65*np.ones((Ni,1)))).flatten()
d = np.vstack((8-6*re**2,2*np.ones((Ni,1)))).flatten()
S = np.hstack((0.5*np.random.rand(Ne+Ni,Ne),-1*np.random.rand(Ne+Ni,Ni)))
v = -65*np.ones(Ne+Ni)
u = b*v
firings = []
T = 1000

for t in range(1,T+1):
    I = np.concatenate((5*np.random.randn(Ne), 2*np.random.randn(Ni)))
    fired = np.where(v>=30)[0]
    if fired.size > 0:
        firings.extend([[t,idx] for idx in fired])
        v[fired] = c[fired]
        u[fired] += d[fired]
        I += np.sum(S[:, fired], axis=1)
    v += 0.5*(0.04*v**2 + 5*v +140 - u + I)
    v += 0.5*(0.04*v**2 + 5*v +140 - u + I)
    u += a*(b*v - u)

f = np.array(firings,dtype=float)
plt.figure(figsize=(10,6))
plt.plot(f[:,0],f[:,1],'.',markersize=2,color='black')
plt.xlabel('Time (ms)')
plt.ylabel('Neuron Number')
plt.title('Neuron Network Simulation, Size = 1000')
plt.show()
```

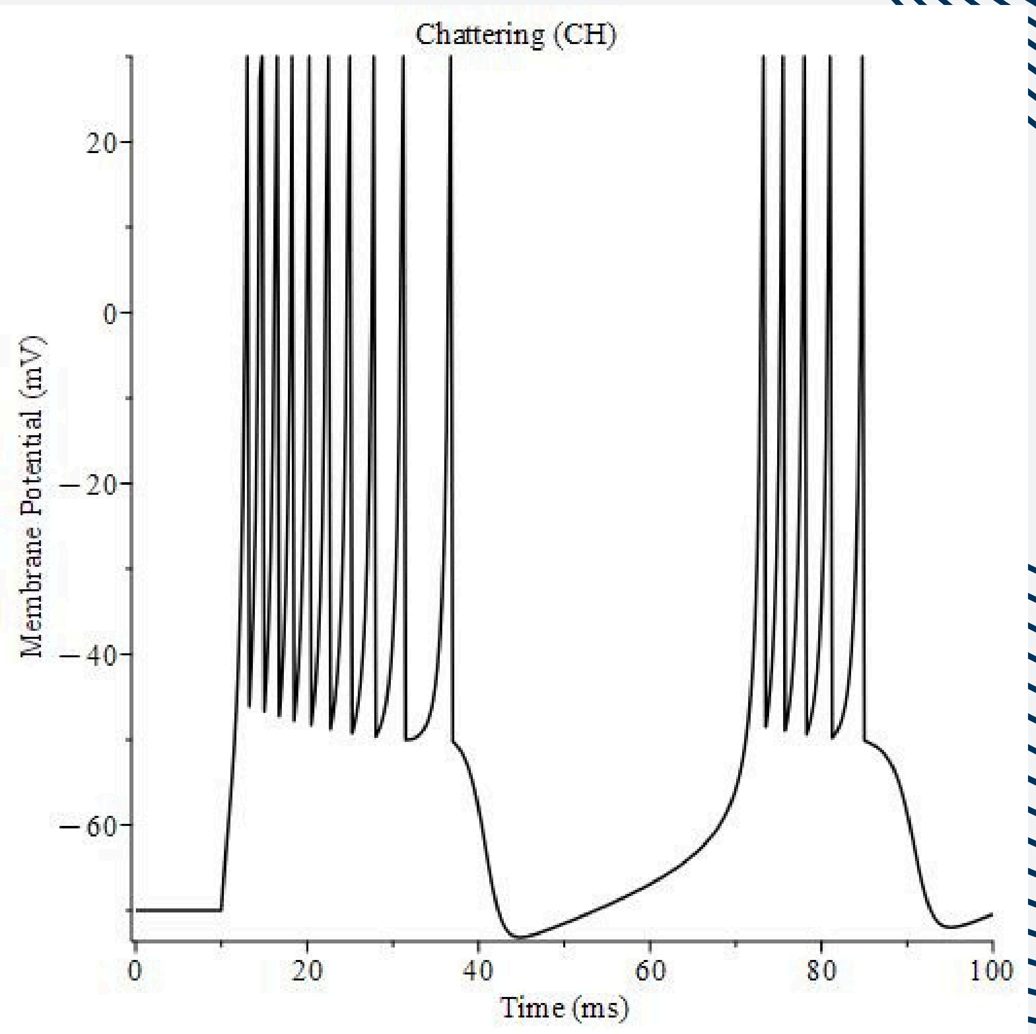
RESULTS - PARAMETER VALUES

Spiking Pattern	a	b	c	d
Regular Spiking (RS)	0.02	0.2	-65	8
Intrinsically Bursting (IB)	0.02	0.2	-55	4
Chattering (CH)	0.02	0.2	-50	2
Fast Spiking (FS)	0.1	0.2	-65	2
Thalamo-Cortical (TC)	0.02	0.25	-65	0.05
Resonator (RZ)	0.1	0.26	-65	8
Low-Threshold Spiking (LTS)	0.02	0.25	-65	2

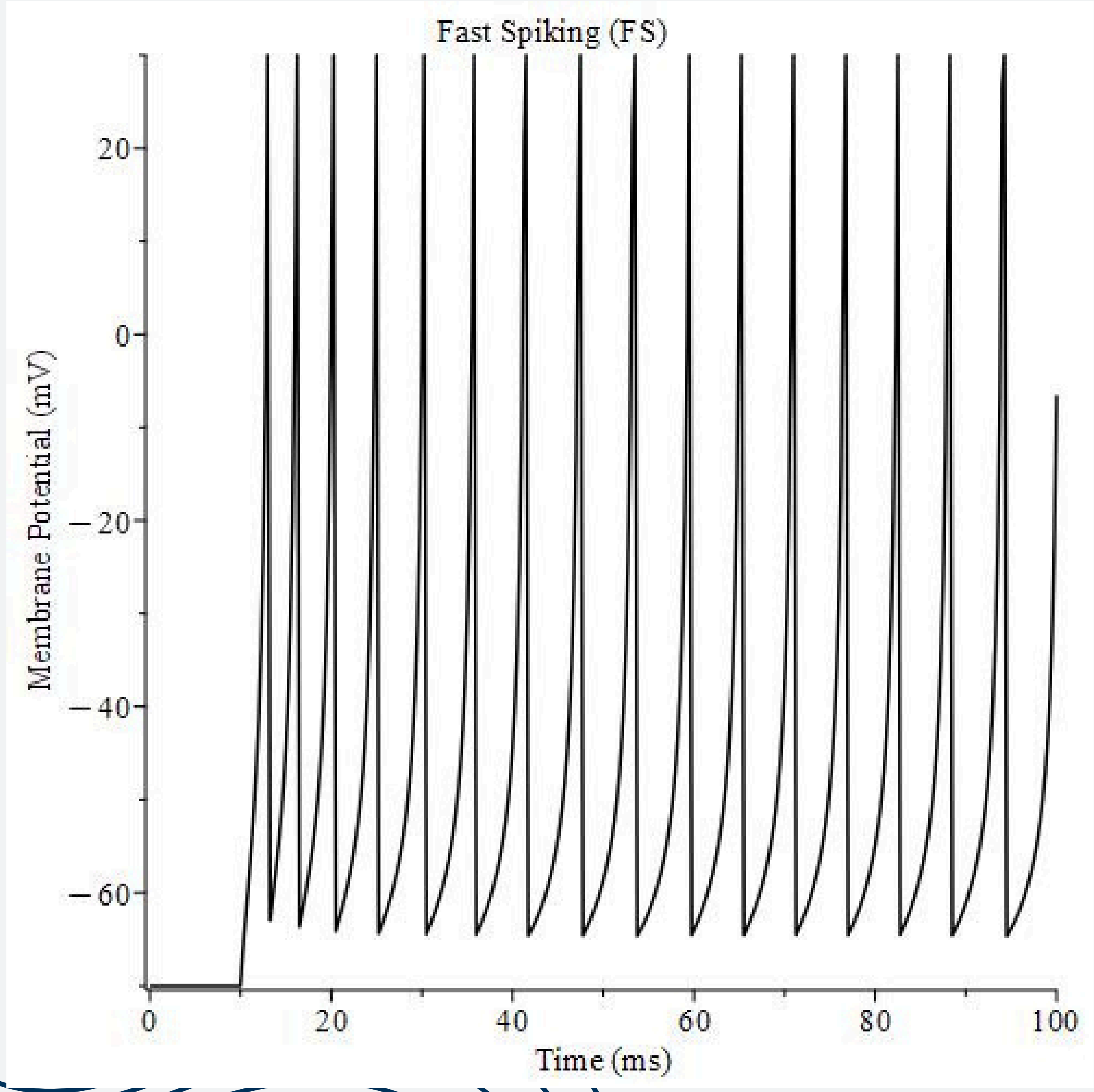
RESULTS - EXCITATORY NEURONS

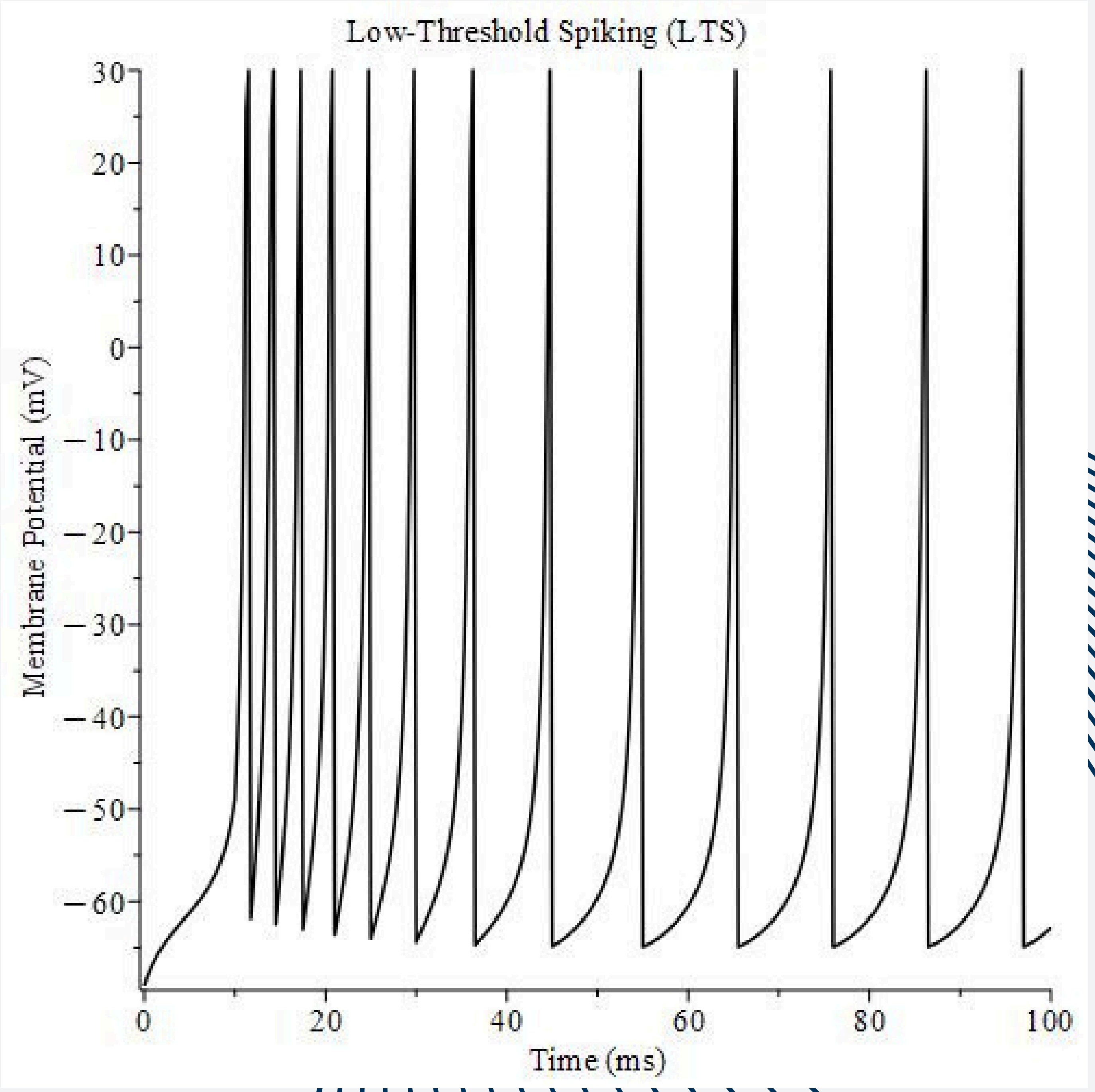




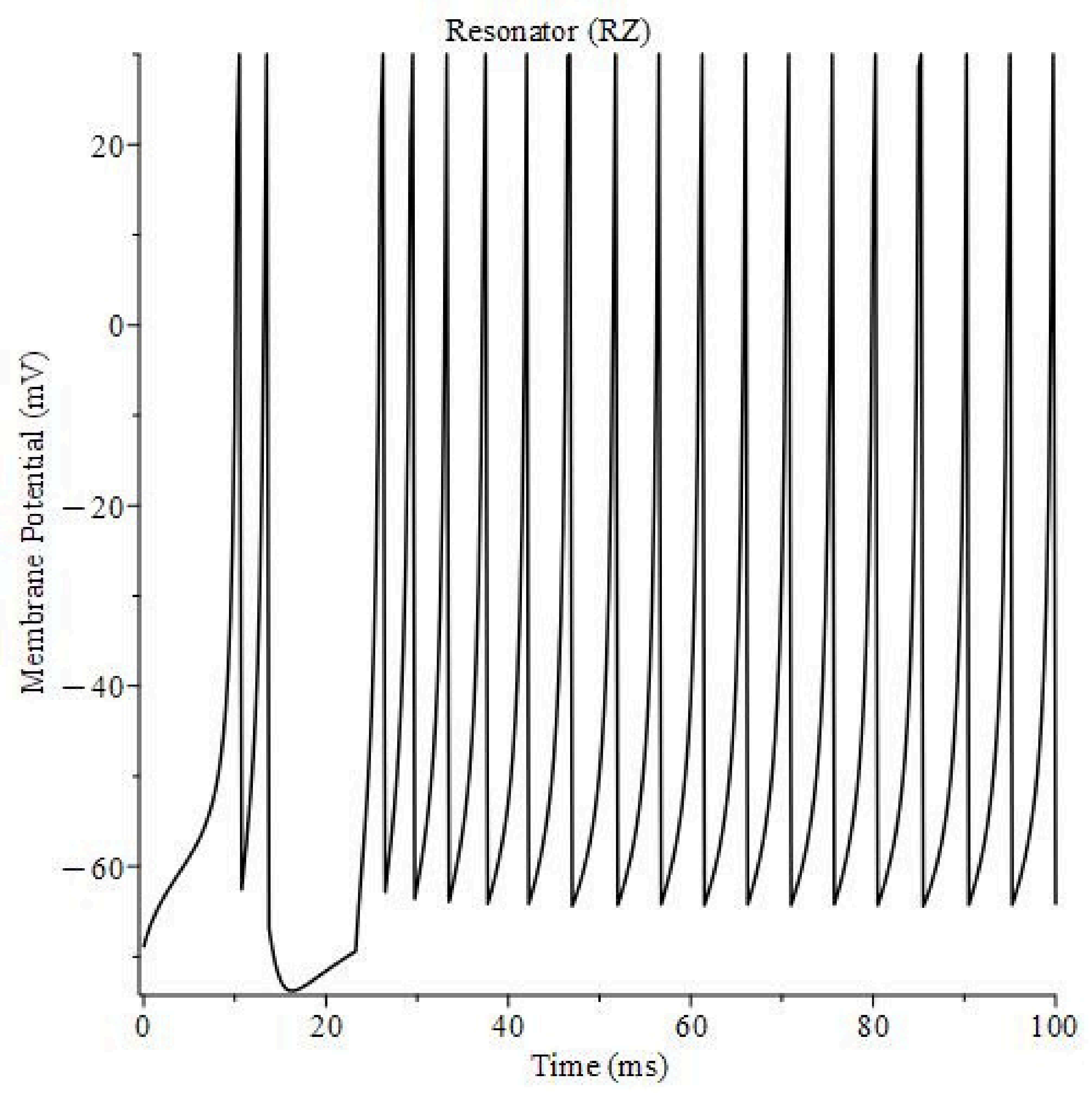


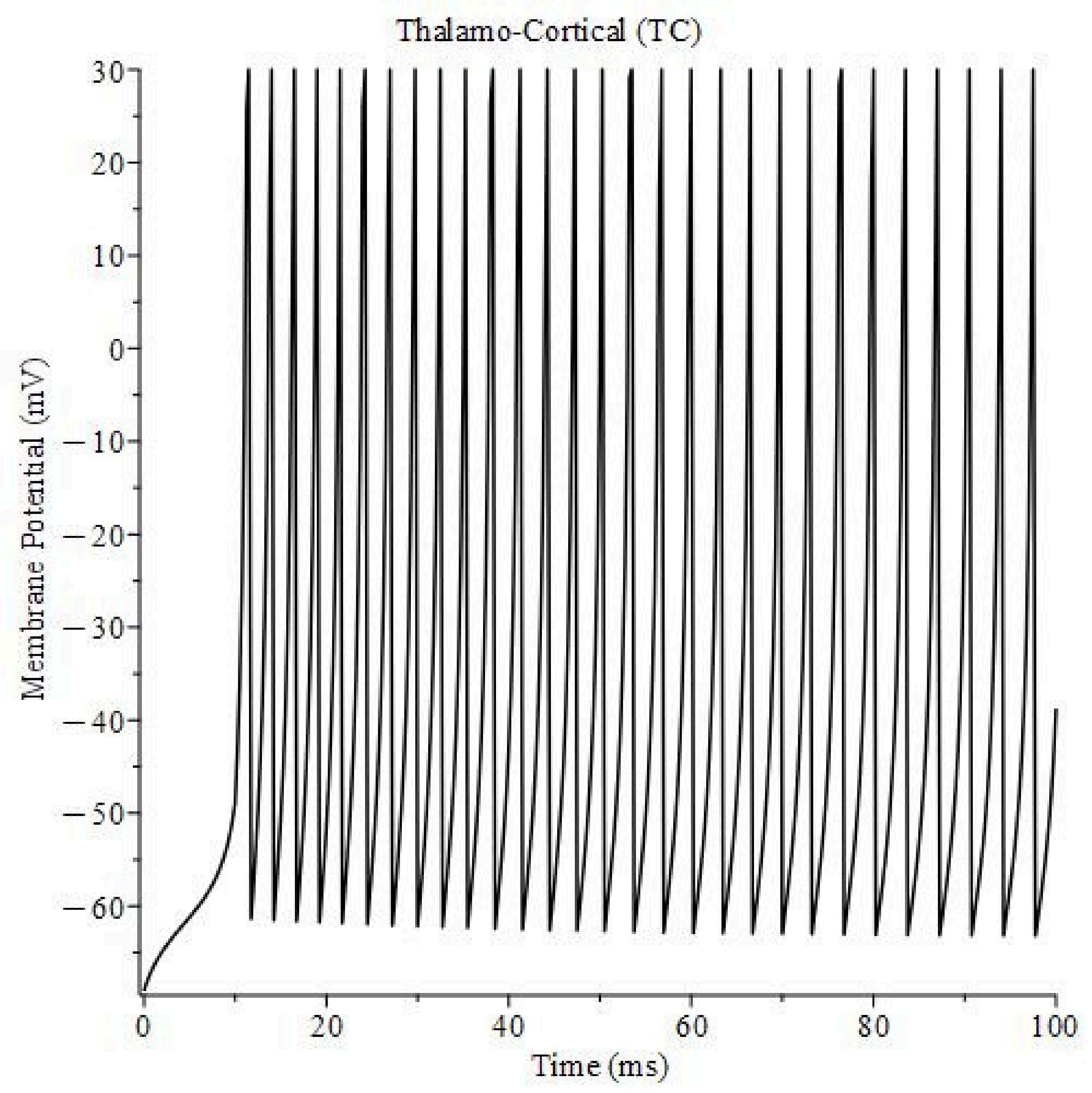
RESULTS - INHIBITORY NEURONS



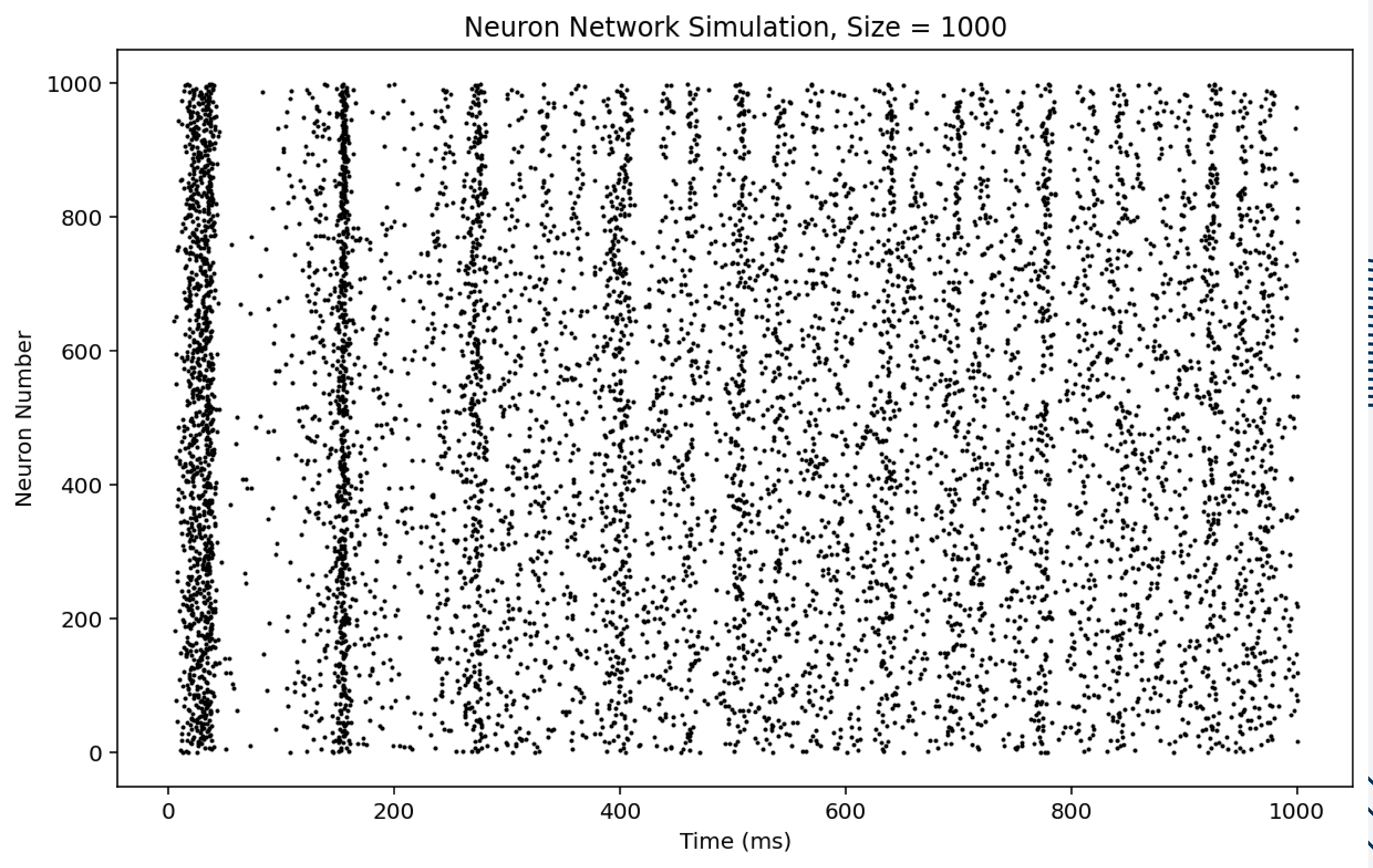


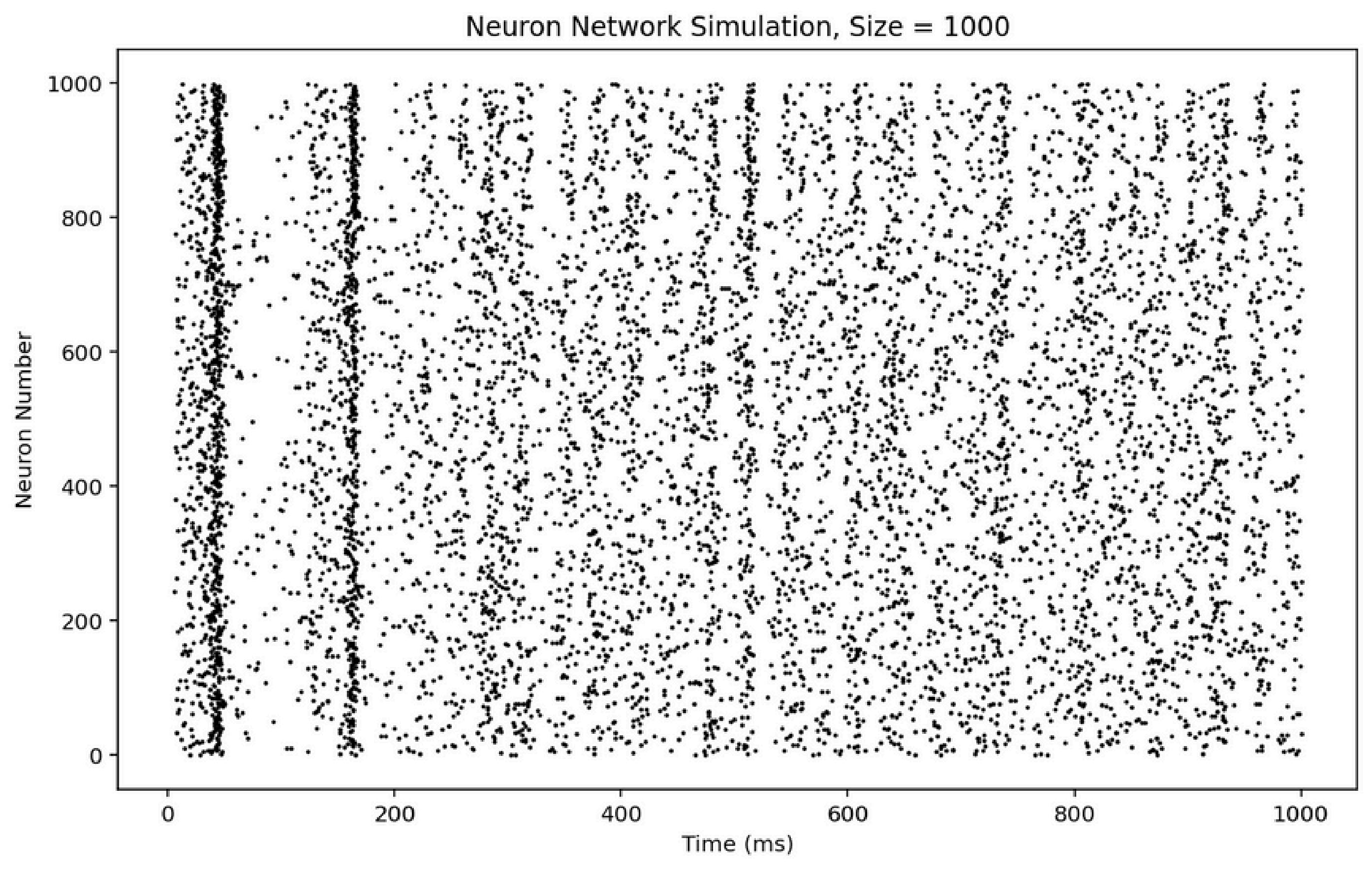
RESULTS - OTHER TYPES AND PATTERNS



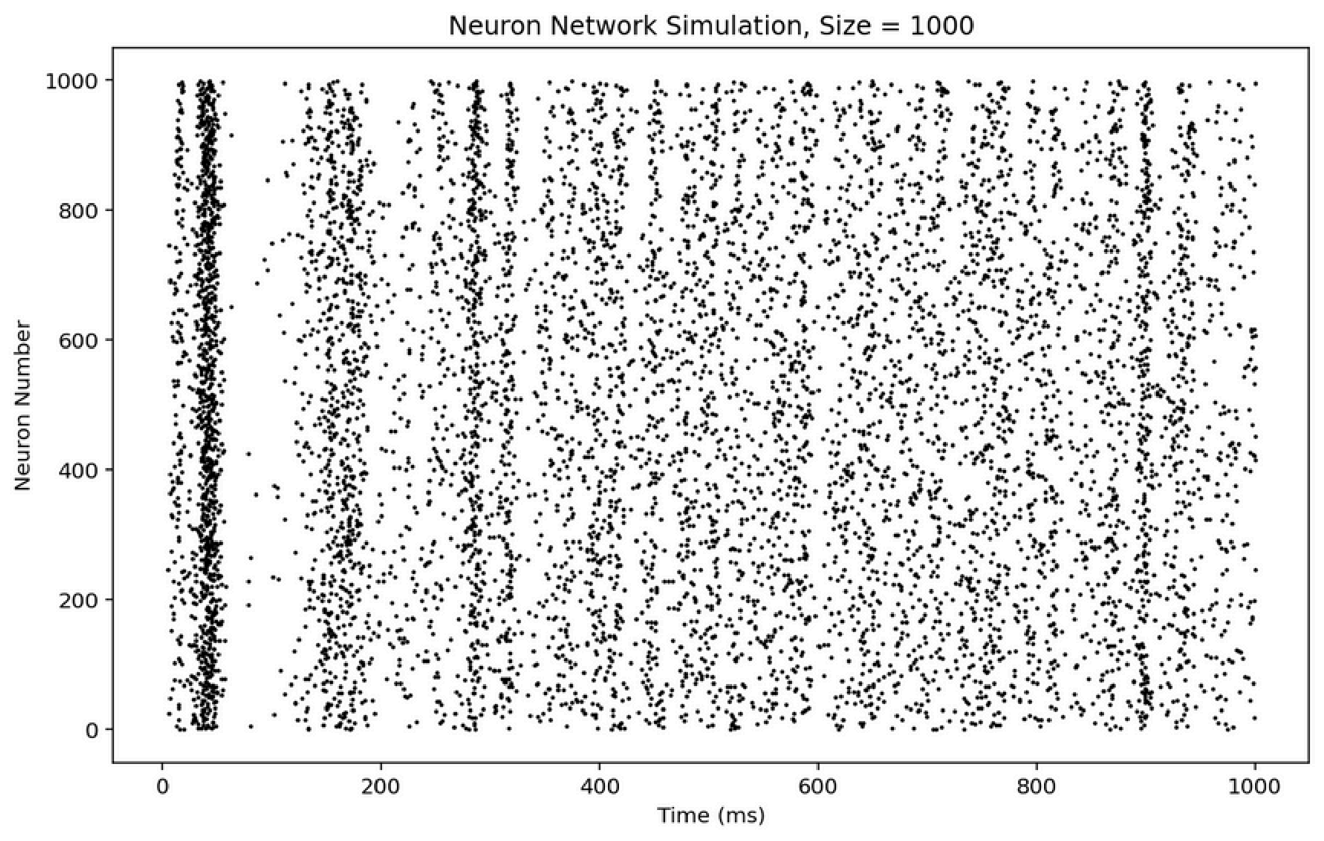


RESULTS - NEURON NETWORK SIMULATIONS

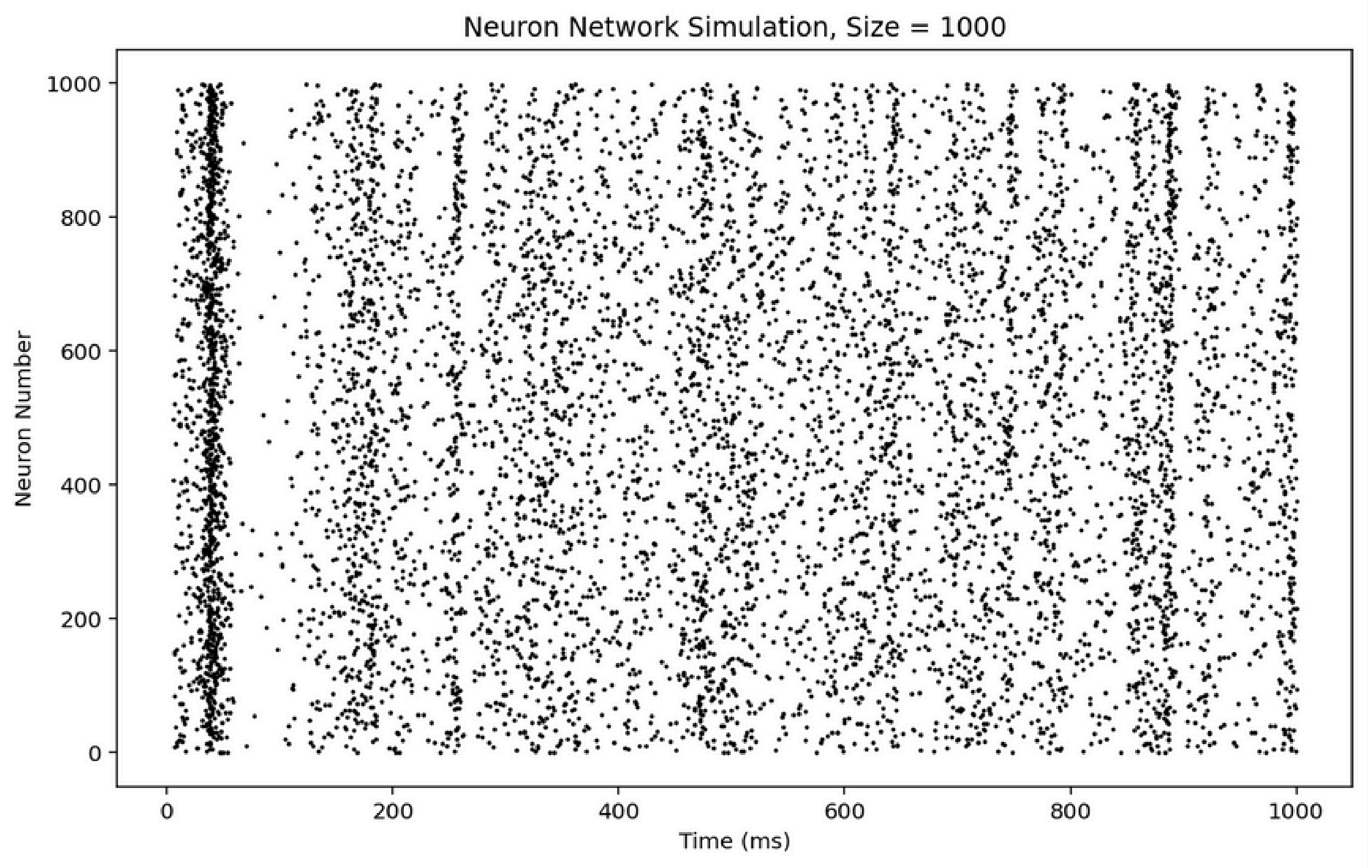




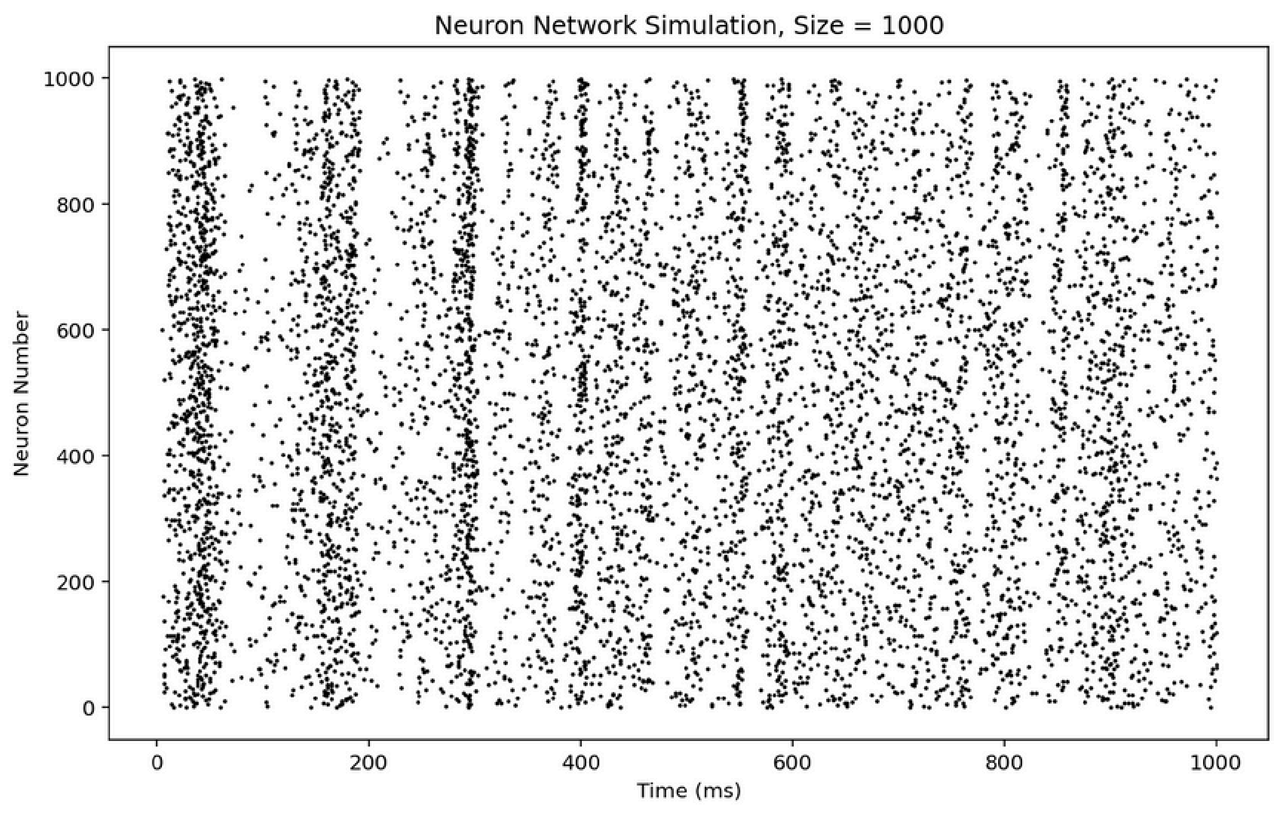
NE = 805, NI = 195



NE = 810, NI = 190



NE = 795, NI = 205



NE = 790, NI = 210

CONCLUSION

The analysis of both single and collective neuron spiking behavior, made computationally possible by Izhikevich's model, can reveal a lot of valuable information about the functions of a neuron or network. Changes in spiking dynamics of individual neurons is thought to relate to the ever-shifting topology of neuron networks, and even entire brain areas (Yang et al., 2025). From the change in behavior of just one neuron, entire neural processes can be elucidated. The neuron network simulation is helpful in that it reveals how an adjustment in the ratio of inhibitory to excitatory neurons is displayed in the of size and distribution of alpha and gamma brain waves. Interestingly, disruption to brain wave patterns is often indicated in many neuro-pathologies, such as Alzheimer's Disease, Depression, and Tinnitus (Guan et al., 2022). Trial therapies for these and other disease of the central nervous system often include using electrical stimulation to increase the frequency of gamma waves.