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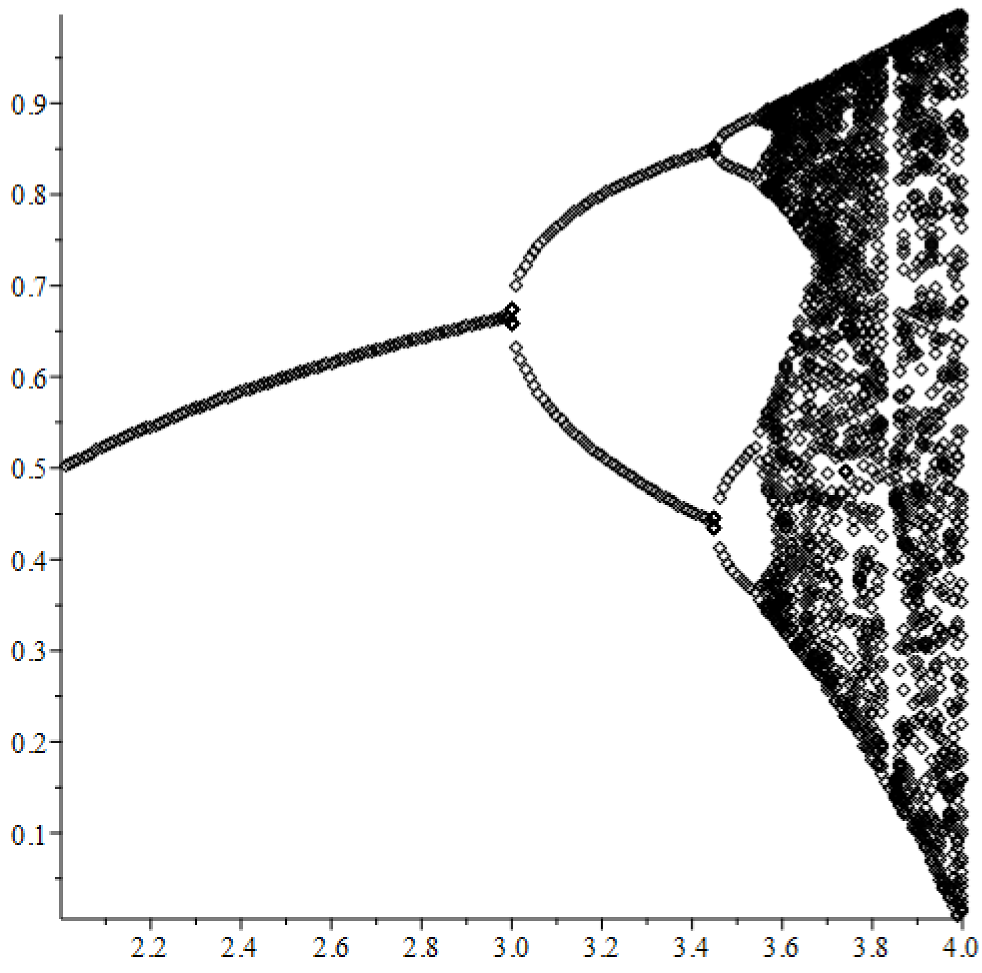
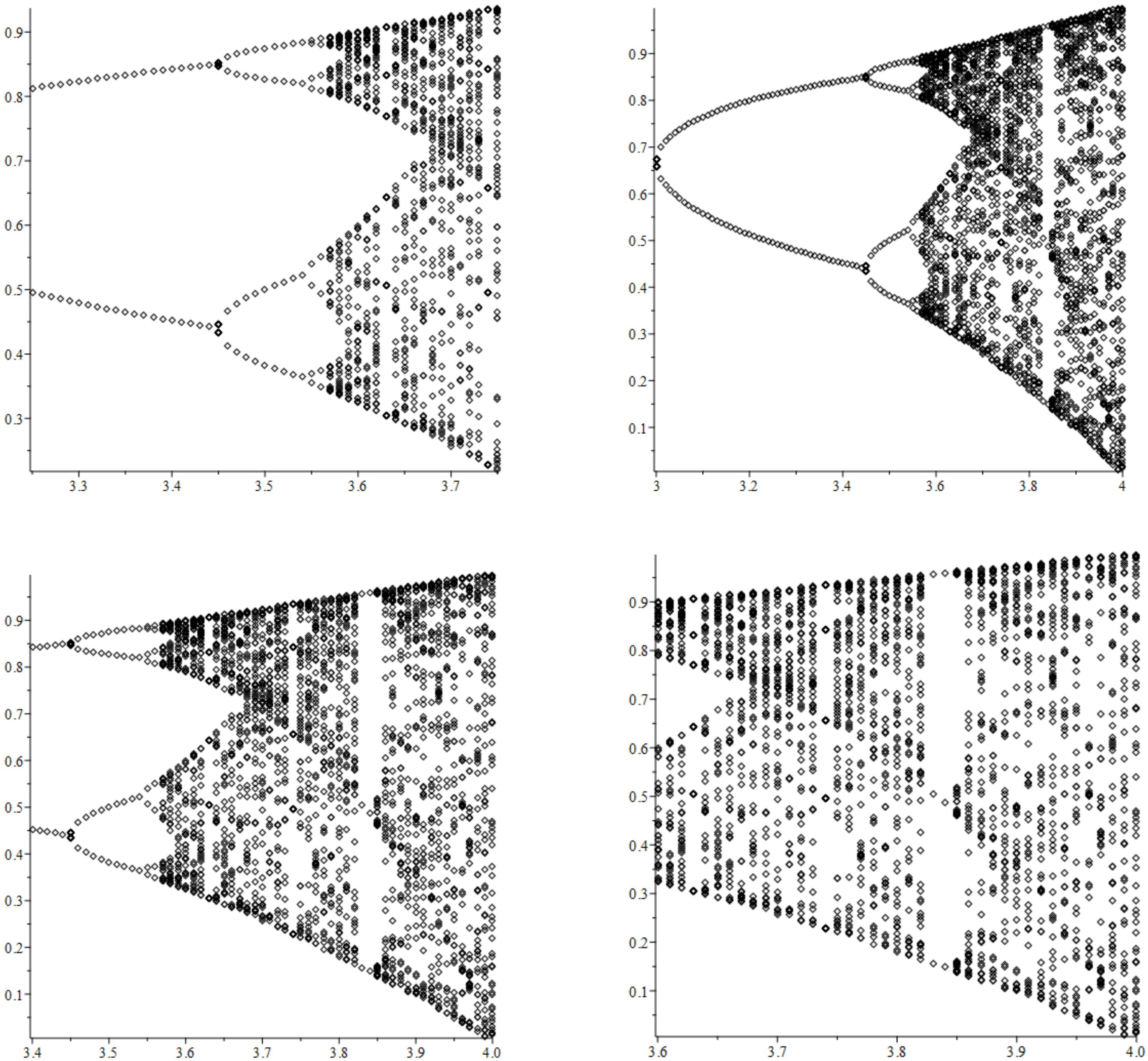
Dynamical Models in Biology
Project 3

Our project was to look into the behavior of the discrete logistic equation. The discrete logistic equation is notable for its sensitivity to different parameters. Depending on the values of coefficients, it may have a single stable state, oscillate between multiple values, or descend into chaos. Another important characteristic is that of period doubling, when the function transitions from oscillating between n many values into oscillating between $2n$ values. These graphs display such behavior.



The function that we are working with here is $kx(1 - x)$. The x-axis represents different values for k while the dots represent values that are output from the function for 64 high values of x, using the `Orb` function in `DMB.txt`. You can see clearly that at 3, there ceases to be steady states. Instead, the function oscillates between 2 values, and then 4, and eventually descends into chaos.



A closer look at certain parts of the graph

We also investigated the behavior of a more general form of the equation. Using the Maple function `F := proc(A,k,x) local i: x-mul(x-A[i],i=1..nops(A))/k end`, we experimented with the oscillating values produced when varying the value of `k`. One example is `F([5,6,7], k, x)`, representing $x - \frac{(x-5)(x-6)(x-7)}{k}$. Such a function does not have steady stable states until $k > 1$ but such states can be estimated based on the steady stable states of the function composed with itself. Here is an example, with the same function as before named `example0`.

```
showExample(example0, k, x)
```

0.81, {4.856042094, 5.291931988, 6.708068012, 7.143957906}

0.82, {4.857814204, 5.282078272, 6.717921728, 7.142185796}

0.83, {4.859767634, 5.272078197, 6.727921803, 7.140232366}

...

0.97, {4.924711463, 5.097916544, 6.902083456, 7.075288537}

0.98, {4.936817660, 5.078239016, 6.921760984, 7.063182340}

0.99, {4.953757058, 5.053757057, 6.946242943, 7.046242942}

1.01, {5., 7.}

[5., 7.]

Here, the values before the comma represent the value of k. The values in the braces show the 4 steady states of the function composed with itself. Those steady states converge to the eventual actual steady state the function reaches once $k \geq 1$, as shown in the printed list. Such an example is notable for (eventually) having 2 steady states. In our experimentation, that only happens when the values in the input list are consecutive and the list only includes 3 terms. The only time we noticed a transition from 4 elements in the set to 2 was when there were 5 consecutive elements in the input. For the following example, example, the function is $x - \frac{(x-8)(x-9)(x-10)(x-11)(x-12)}{k}$. Again, the increasing number is for the value of k.

$$1.21, \{9.329251201, 9.588185844, 10.41181416, 10.67074880\}$$
$$1.22, \{9.336070871, 9.580801210, 10.41919879, 10.66392913\}$$
$$1.23, \{9.343320972, 9.572987630, 10.42701237, 10.65667903\}$$
$$1.24, \{9.351092416, 9.564654188, 10.43534581, 10.64890758\}$$
$$1.25, \{9.359513368, 9.555672707, 10.44432729, 10.64048663\}$$
$$1.26, \{9.368774928, 9.545852080, 10.45414792, 10.63122507\}$$
$$1.27, \{9.379185321, 9.534884075, 10.46511592, 10.62081468\}$$
$$1.28, \{9.391304693, 9.522208538, 10.47779146, 10.60869531\}$$
$$1.29, \{9.406385744, 9.506572761, 10.49342724, 10.59361426\}$$
$$1.30, \{9.429059721, 9.483345488, 10.51665451, 10.57094028\}$$
$$1.31, \{9.458527608, 10.54147239\}$$
$$1.32, \{9.462723943, 10.53727606\}$$
$$1.33, \{9.466944638, 10.53305536\}$$
$$1.34, \{9.471190328, 10.52880967\}$$

It's unclear what occurs between $k = 1.30$ and $k = 1.31$. Here is $x - \frac{(x-1)(x-2)(x-7)(x-8)}{k}$.

```
showExample(example5, k, x)
```

$$12.34, \{1.548968673, 2.254549772\}$$
$$12.35, \{1.549847625, 2.254400450\}$$

…

$$14.96, \{1.950135786, 2.046889666\}$$
$$14.97, \{1.956970990, 2.040798095\}$$
$$14.98, \{1.965019473, 2.033493249\}$$
$$14.99, \{1.975408705, 2.023847656\}$$
$$15.01, \{2.\}$$

For this example, because the input is not consecutive, it only results in one stable steady state. However, the estimated states still approach it. Several more examples are available in the attached code.