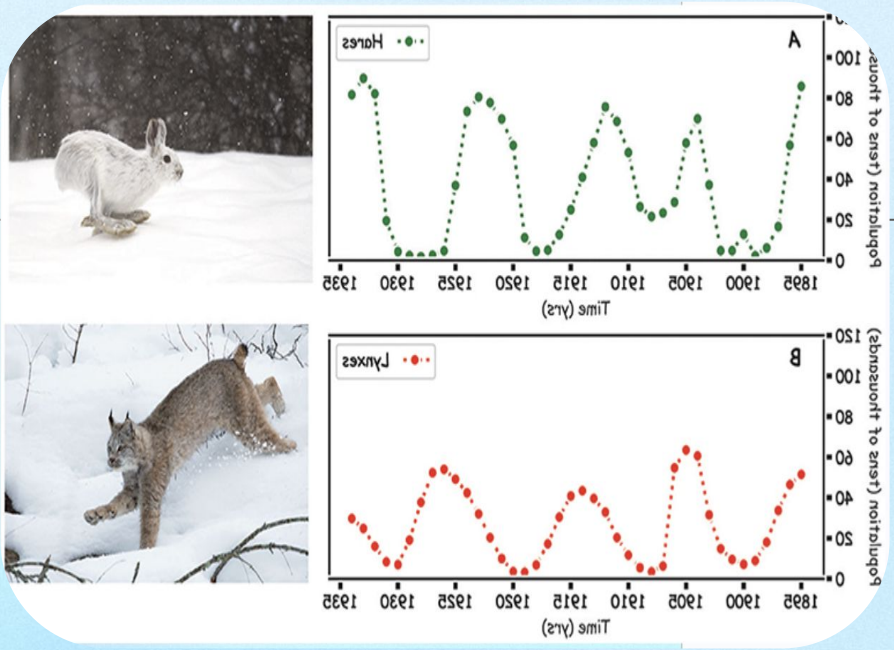
Predator/Prey Modeling



Rachel Adelman, Sophie Droppa, Anna Janik, Sydney Yao

Nicholson-Bailey Host-Parasite Equations

- Prey equation→ H(t)
- Predator equation → P(t)
- Prey carrying capacity→ K
- Strength of Interaction between Predator/Prey→ a

$$H_{t+1} = H_t \exp[r(1 - H_t/K) - aP_t]$$

$$P_{t+1} = \alpha H_t[1 - \exp(- aP_t)]$$

Stability

- r= prey reproductive rate
- q = depression of prey equilibrium (how much predators suppress it)

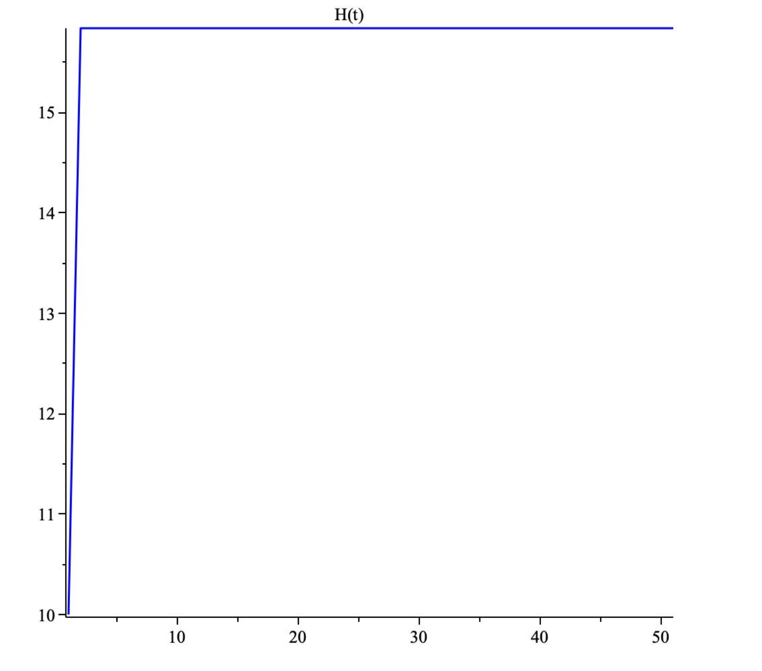
Beddington

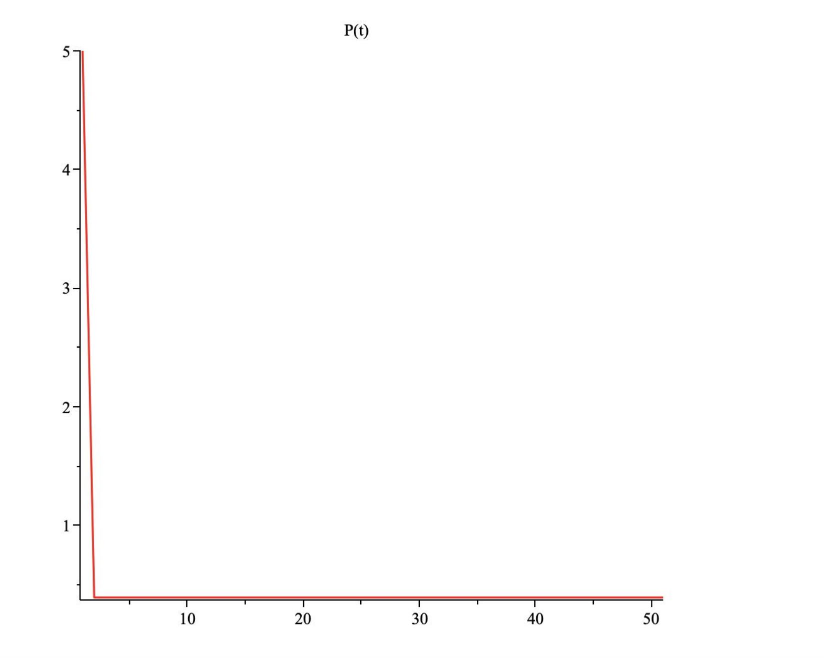
$$\lambda^2 - \lambda(1 - r + \varphi) + (1 - rq)\varphi + r^2 q(1 - q) = 0$$

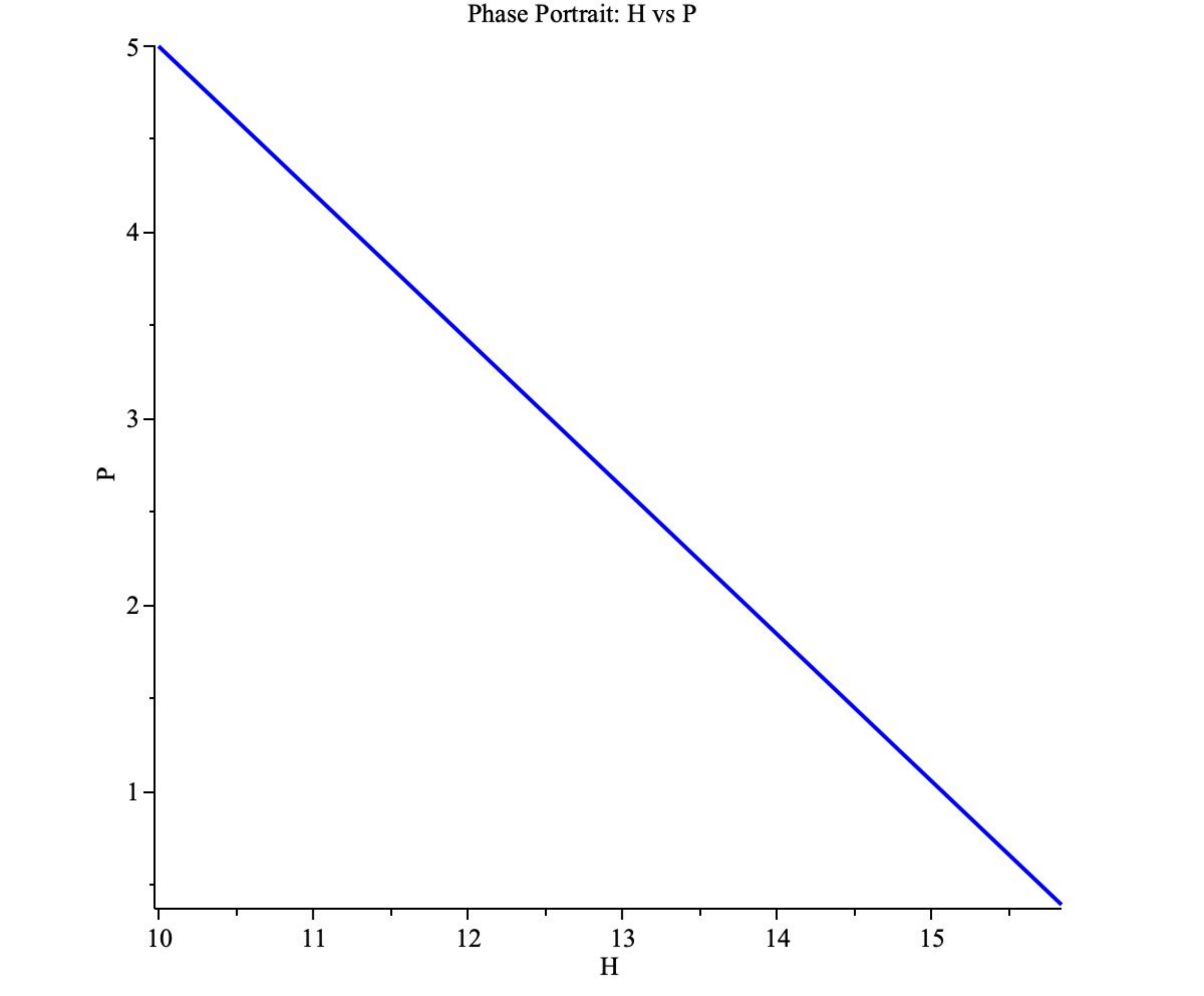
where $\varphi = r(1 - q)/\{1 - \exp[-r(1 - q)]\}$

- If both λ's have |λ| < 1 → the equilibrium is stable.
- If |λ| > 1 → equilibrium is unstable → oscillations or chaos.

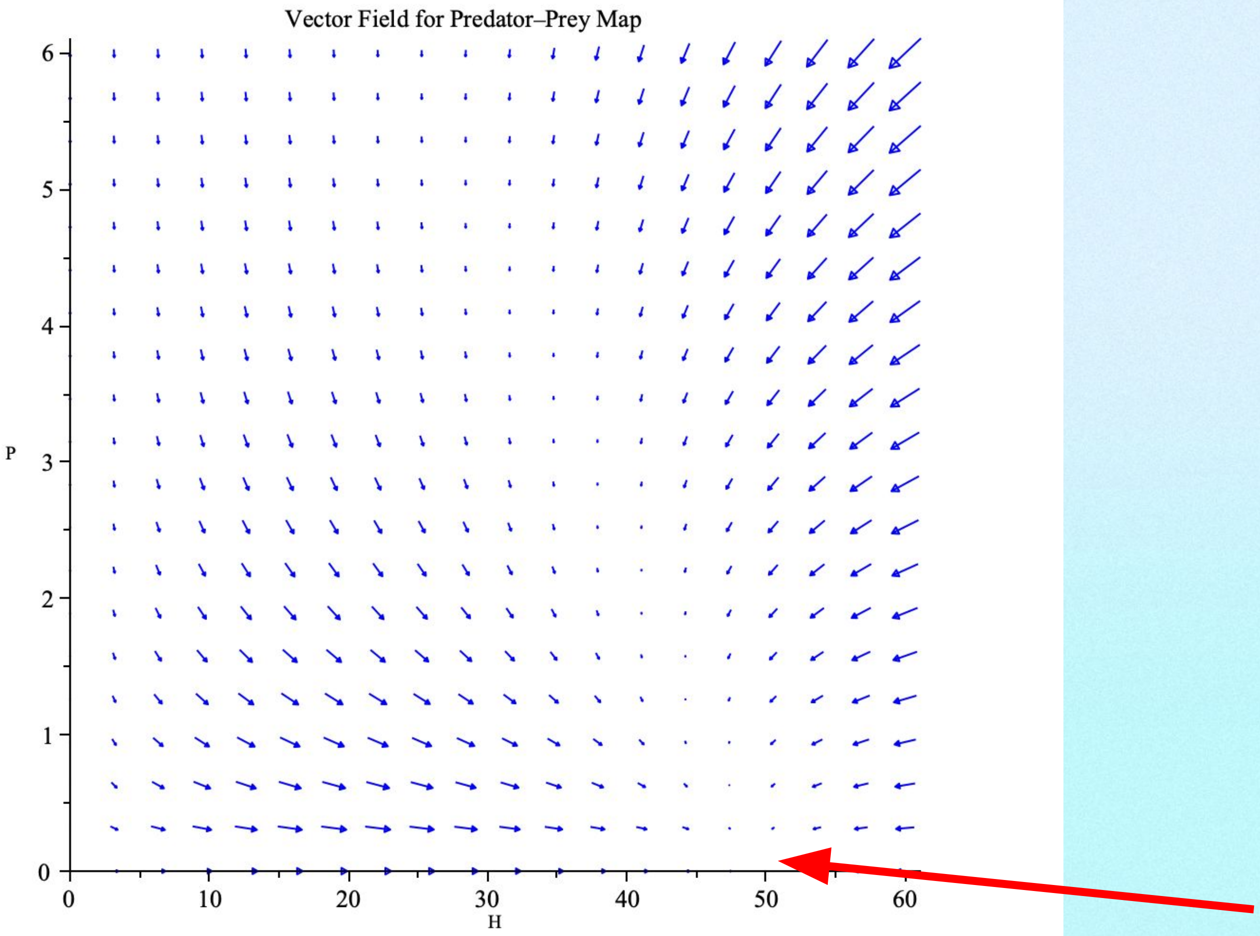
Case 1: r=1.2 a=.10,K=50,H=10,P=5





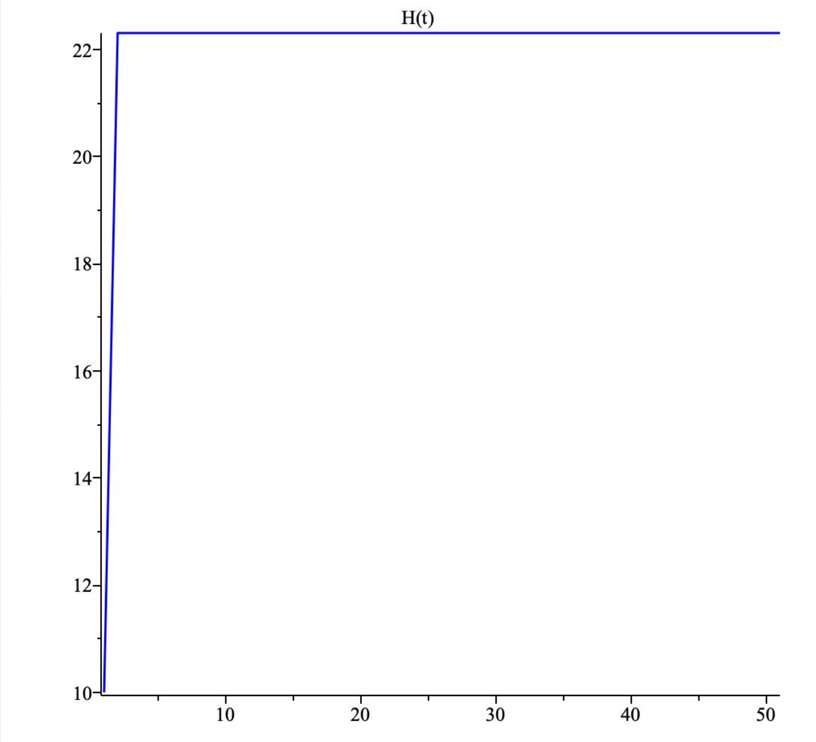


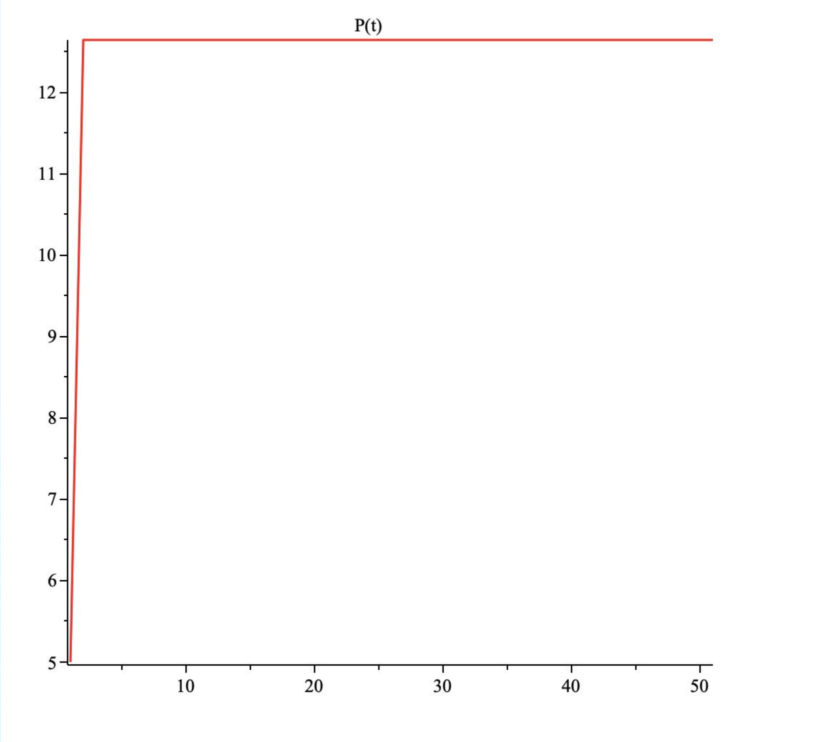
Phase Portrait: H vs P
P
H
5
4
3
2
1
10
11
12
13
14
15

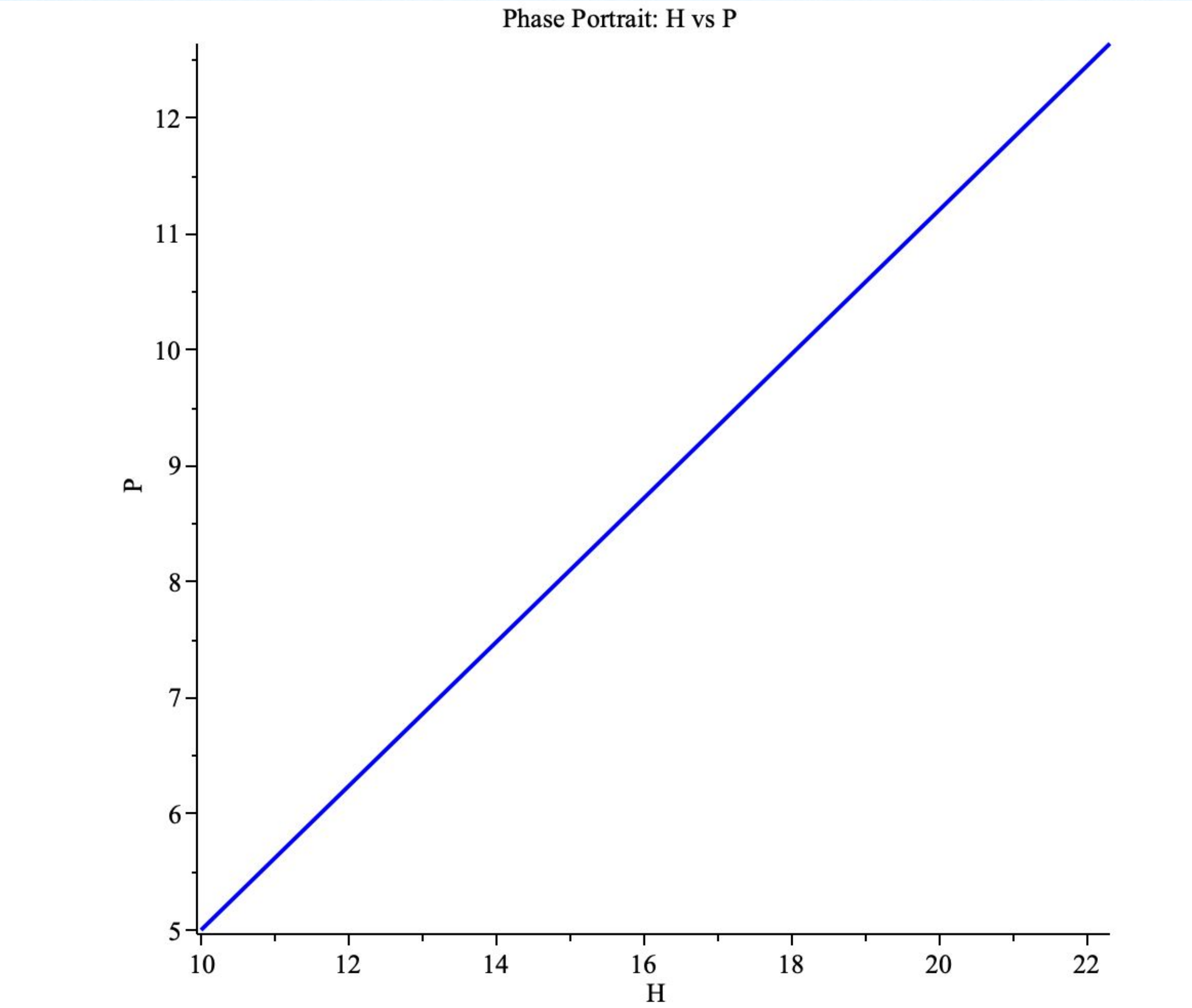


Vector Field for Predator–Prey Map
P
H
0
10
20
30
40
50
60
0
1
2
3
4
5
6

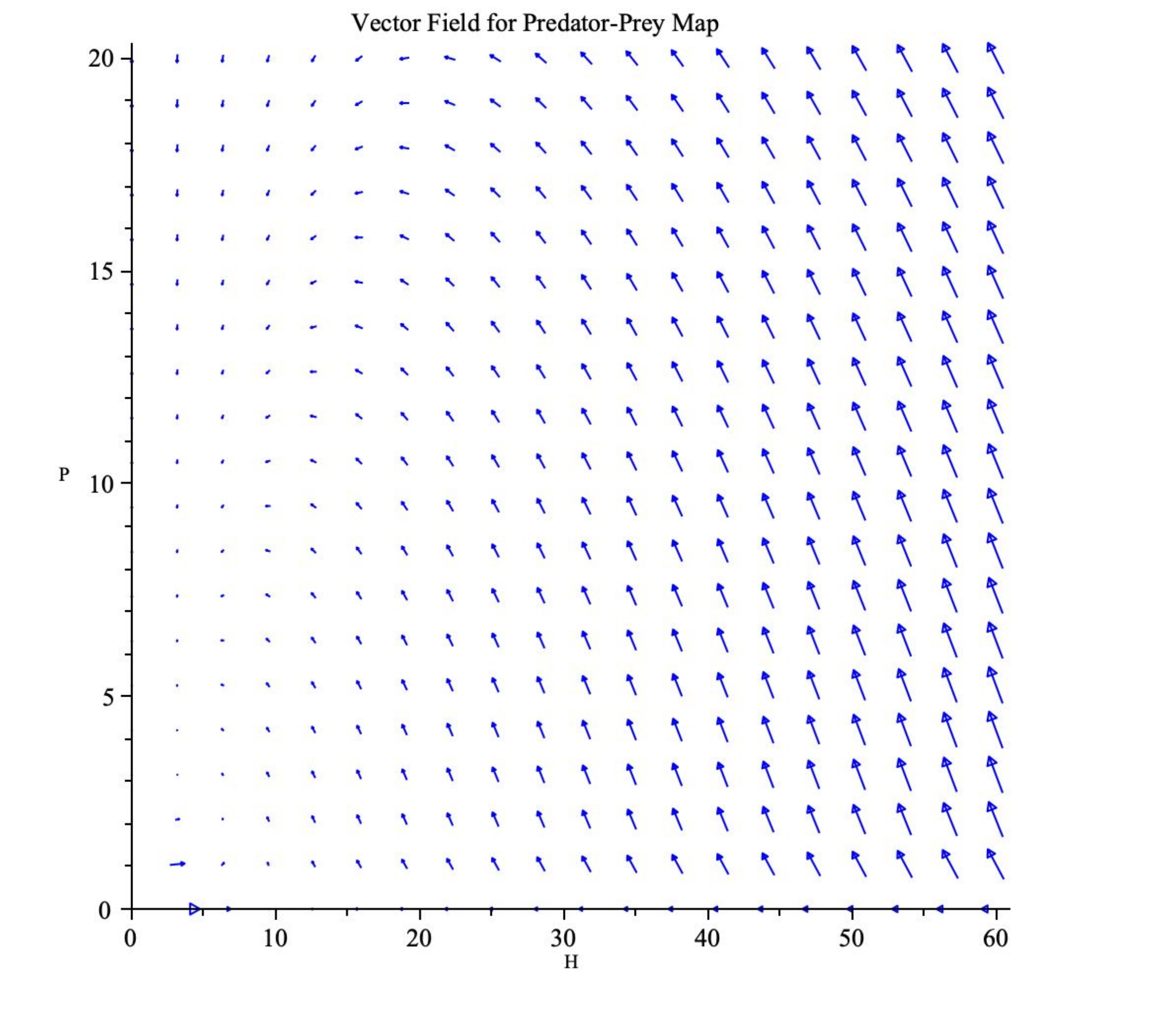
Case 2: r=.5, a=.2,K=50,H=100,P=5





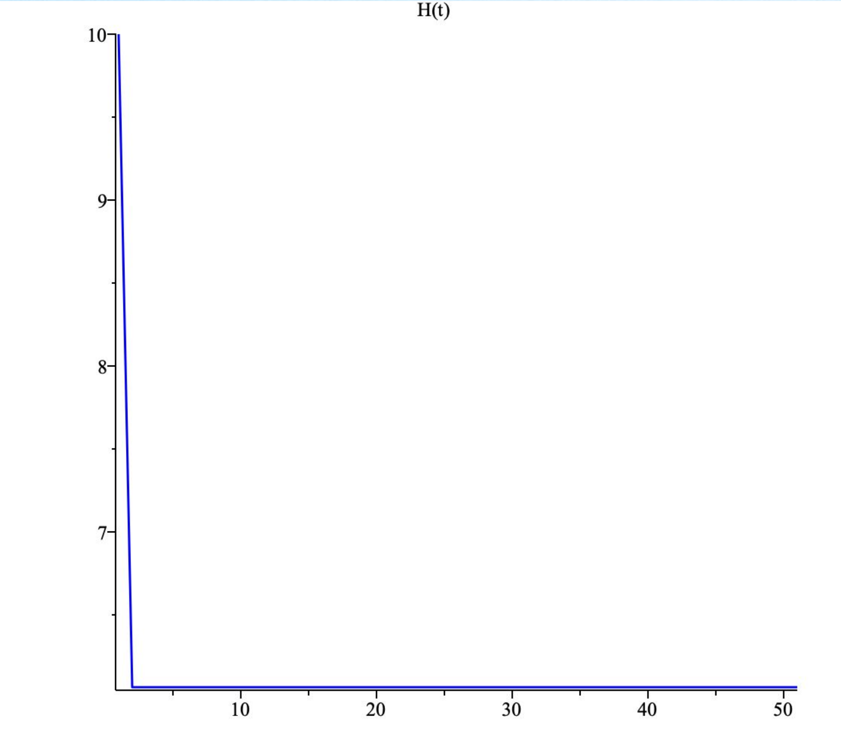


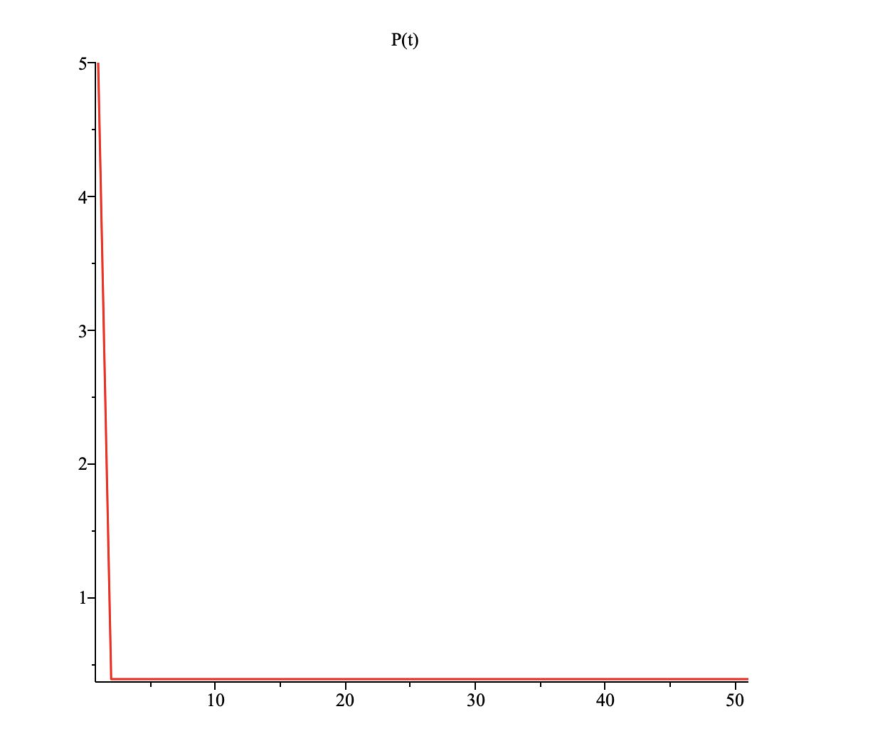
Phase Portrait: H vs P
P
H
12
11
10
9
8
7
6
5
10
12
14
16
18
20
22

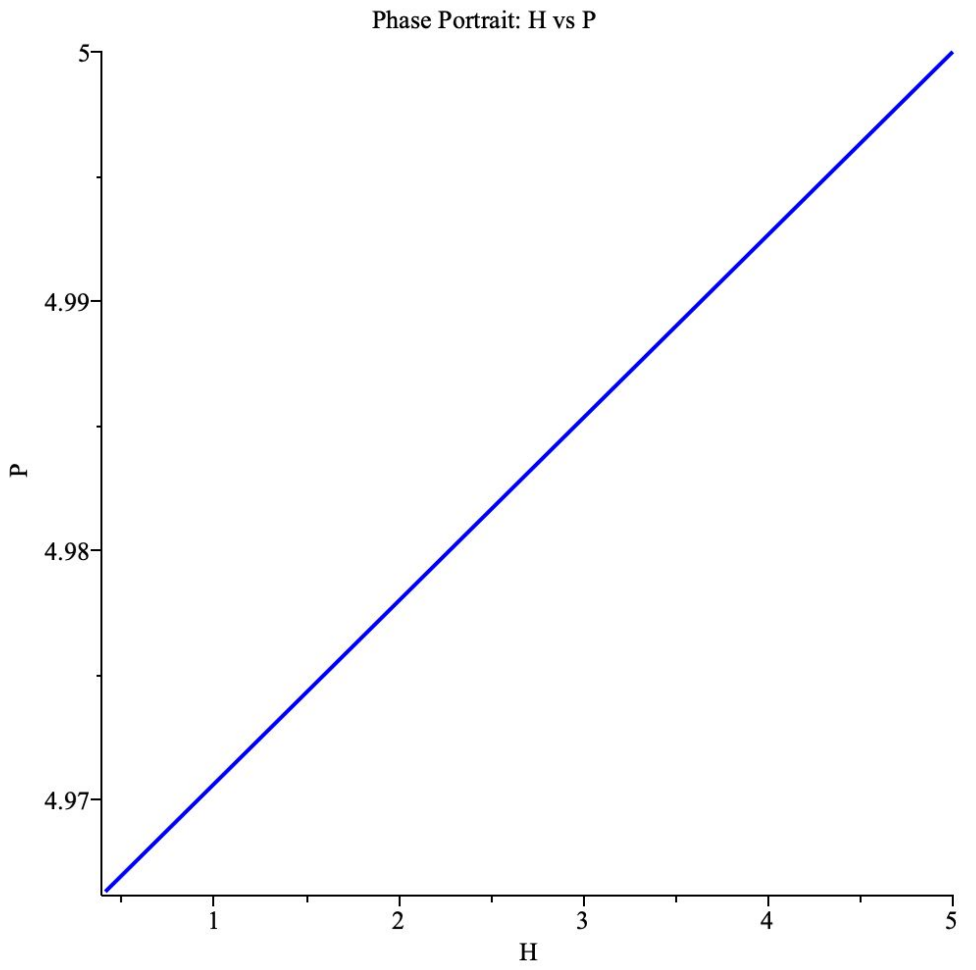


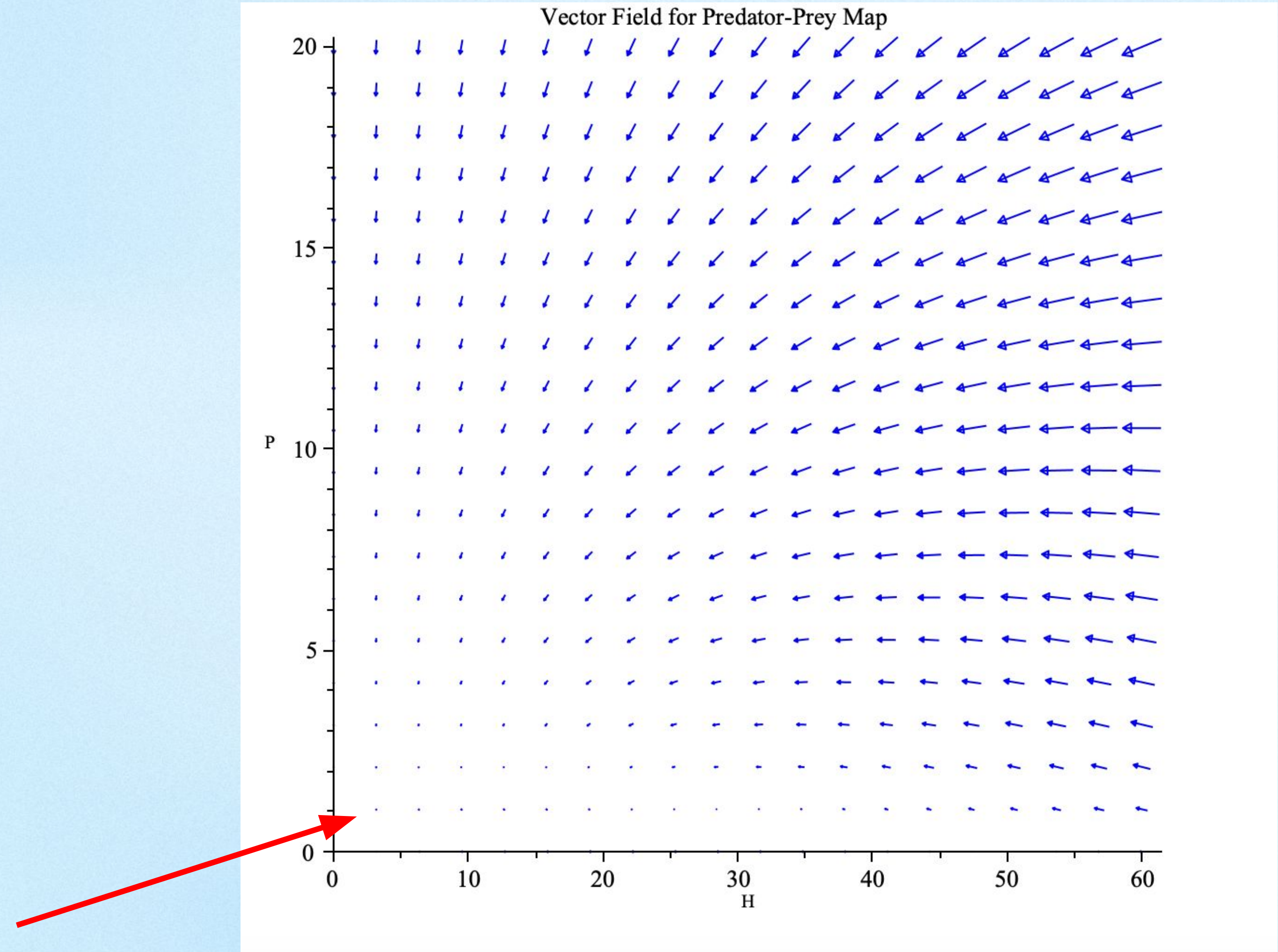
Vector Field for Predator-Prey Map
P
H
0
5
10
15
20
0
10
20
30
40
50
60

Case 3: r=5, a=1,K=10,H=5,P=5






Phase Portrait: H vs P
5
4.99
4.98
4.97
P
1
2
3
4
5
H


Vector Field for Predator-Prey Map
P
H
0
5
10
15
20
0
10
20
30
40
50
60

Conclusion

- Difference equations reveal full predator–prey dynamics.
- Complex behavior can arise without environmental randomness.
- Biology alone can create stability, cycles, or chaos.
- Helps explain unpredictable population fluctuations.
- Useful for forecasting predator-prey relationships