1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3} \quad , \quad n \geq 3$$

subject to the initial conditions

$$x_0 = 1, x_1 = 2, x_2 = -1 \quad .$$

not sure if you wanted next 6 terms OR the first 6 so I did them all.

$X_0 = 1$
$X_1 = 2$
$X_2 = -1$
$X_3 = 2$
$X_4 = -2$
$X_5 = 3$
$X_6 = -3$
$X_7 = 5$
$X_8 = -4$

first 6 terms

next six terms

$X_3 = X_2 + 2X_1 - X_0$
$X_3 = -1 + 4 - 1$
$X_3 = 2$

$X_4 = X_3 + 2X_2 - X_1$
$X_4 = 2 + 2(-1) - 2$
$X_4 = -2$

$X_5 = X_4 + 2X_3 - X_2$
$X_5 = -2 + 2(2) + 1$
$X_5 = 3$

$X_6 = X_5 + 2X_4 - X_3$
$X_6 = 3 + 2(-2) - 2$
$X_6 = -3$

$X_7 = X_6 + 2X_5 - X_4$
$X_7 = -3 + 2(3) + 2$
$X_7 = 5$

$X_8 = X_7 + 2X_6 - X_5$
$X_8 = 5 + 2(-3) - 3$
$X_8 = -4$

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2} \quad , \qquad K = 2$$

with initial conditions

$$x_0 = 0, x_1 = 1 \quad .$$

$z^2 = 5z - 6$
$z^2 - 5z + 6 = 0$
$(z_1 - 2)(z_2 - 3) = 0$
$z_1 = 2$
$z_2 = 3$

general sol'n:

$X_n = C_1 z_1^n + C_2 z_2^n$

$X_n = C_1 3^n + C_2 2^n$
$X_0 = 0 = C_1 + C_2$
$X_1 = 1 = 3C_1 + 2C_2$

$-2(0 = C_1 + C_2)$
$+ \quad 1 = 3C_1 + 2C_2$
$1 = C_1$

sol'n: $\boxed{X_n = 3^n - 2^n}$

$C_1 = 1, C_2 = -1$

3. (Corrected Sept. 6, 2025, thanks to Caroline Hill [who won a dollar].)

In a certain species of animals, only one-year-old, two-year-old are fertile.

The probabilities of a one-year-old, two-year-old, female to give birth to a new female are p_1, p_2, respectivley.

Assuming that there were c_0 females born at $n = 0$, c_1 females born at $n = 1$ Set up a recurrence that will enable you to find the **expected** number of females born at time n.

In terms of c_0, c_1, p_1, p_2, how many females were born at $n = 4$?

one-year olds at time $n = C_{n-1}$
two-year olds at time $n = C_{n-2}$

$$C_n = P_1 C_{n-1} + P_2 C_{n-2}$$

given $\underset{=0}{C_0}, \underset{=1}{C_1}$, $\rightarrow$ find C_3 and C_4

$$C_3 = P_1 C_2 + P_2 C_1$$

$$C_4 = P_1 C_3 + P_2 C_2$$

$$C_4 = P_1 (P_1 C_2 + P_2 C_1) + P_2 C_2$$

$$C_4 = P_1^2 C_2 + P_1 P_2 C_1 + P_2 C_2$$

$$C_4 = P_1^2 C_2 + (P_1 C_1 + C_2) P_2$$