Basic Concepts Qualifying test

In order to be pass this class with B or up you need to score at least 90 points. In you score between 70 to 90 points you may still pass it with a C (but not with an A or B). If you score less than 70 points, then you failed this class. Please use you own paper. Make sure to have your name. **EXPLAIN EVERYTHING**.

1. (10 points altogether) Consider the function defined on $f : \{1, \ldots, 10\} \to \{1, \ldots, 10\}$, defined by

$$f(x) = x^3 \ (mod \quad 11)$$

(Recall that $a \quad mod \quad b$ means the remainder when you divide a by b).

(a) (5 points) Find all the fixed points. (b) (5 points) Find the trajectory that starts with $x_0 = 2$

2. (10 points altogether) (a) (2 points) Write down the **format** of a continuous-time dynamical system with three quantities (call them $x(t)$, $y(t)$, $z(t)$) (b) (2 points) *Define* an **equilibrium solution**. (c) (2 points) Describe how to find them. (d) (2 points) *Define* **stable equilibrium solution** (e) (2 points) Describe how to find all the stable equilibrium solutions, using calculus (not numerics)

3. (10 points altogether) In our own planet, the force of gravity acting on a particle of mass m, pointing towards the center of the earth is mg where g is Newton's constant. Also in our planet (and universe) the force equals the mass times the second derivative of the position.

(a) (4 points) A ball is dropped (freely, i.e. it is not pushed down or up), at time $t = 0$, from a tower of height 1000 meters. If $x(t)$ is the distance of the particle from the top of the tower at time t set up a differential equation for $x(t)$ and the initial conditions.

(b) (4 points) Solve the initial value problem, and get an explicit formula in terms of t for $x(t)$.

(c) (2 points) Find the time that it would take to reach the ground, (Leave your answer in terms of g).

4. (10 points) A certain particle is moving in such a way that its fifth-derivative with respect to time is 1 m/s^5 At time $t = 0$ its position, and the first four derivatives of the position are 0. How far from the starting point is the particle after 2 seconds?

5. (10 points) In a mini-internet with two web-sites only, the probability of a random surfer, who is currently at site 1 of staying there at the next time-step is $\frac{1}{3}$ and the probability of random surfer, who is currently at site 2 of still staying there at the next time-step is $\frac{1}{5}$. In the long run, what fractions of the times are spent at each site?

6. (10 points) For each of the following equations (or system of equations), and for each of the properties, state whether it is : (i) algebraic or differential or difference (ii) linear or non-linear

(a) $x^3-6x+1=0$ (b) $\{x+2y=6, 6x+y=11\}$ (c) $x''(t)=x'(t)^3+x(t)$ (d) $x''(t)-3x'(t)+2x(t)=0$
(e) $x(n)=x(n-1)+y(n-1) \quad , \quad y(n)=x(n-1)+4y(n-1)$

(f) $x(n)=x(n-1)^2+y(n-1) \quad , \quad y(n)=x(n-1)+4y(n-1)$

7. Consider the transformation from R^2 to R^2 given by

$$[x,y] \to [(1-3x-3y)(2-x-2y),(2-3x-2y)(1-3x-y)] \quad .$$

(a) (5 points) Write down the continuous-time dynamical system with the above underlying transformation.

(b) (5 points) According to `DMB.txt`, the set of equilibrium solutions is $\left\{[0,1],\left[\frac{1}{3},0\right],\left[\frac{4}{3},-1\right]\right\}$.

Also according to `DMB.txt` the set of stable equilibrium solutions is $[\frac{4}{3},-1]$.

If the initial conditions for the above dynamical system are $x(0)=\frac{4}{3}$ and $y(0)=-1$, what is the value of $x(1000)$ **exactly**?

If the initial conditions for the above dynamical system are $x(0)=1.3$ and $y(0)=-0.9$, what is the value of $y(1000)$ **approximately**?

8. (10 points) 1000 chickens lay 1000 eggs in 1000 days. How many eggs do 2000 chickens lay in 2000 days?

9. (5 points) Verify whether or not the function $x(t)=\sin t$ is a solution of the initial value problem differential equation

$$x(t)^2+x'(t)^2=1 \quad , \quad x(0)=0 \quad .$$

10 (5 points) A woman looks at a picture on the wall and says: "Brothers and sisters have I none, but this woman's mother is my mother's daugheter". At whose picture is she looking at.

11. (a) (5 points) Explain how you find the equilibrium solutions for discrete-time and continuous-time dynamical systems. Why are the methods different?

(b) (5 points) Explain how you find the stable equilibrium solutions for discrete-time and continuous-time dynamical systems. Why are the methods different?