Solutions to the Pre-Attendance quiz for Lecture 6

Since no one got it completely, I am posting the question and the full solution to today's "pre-attendance" quiz given at Beck 250 today (Sept. 21, 2021)

1. In a certain population there are are only 3 age groups:

0-year-olds; 1-year-olds; 2-year-olds;

We have the following data

- The probability of survival from 0-year old to 1-year old is 0.95
- The probability of survival from 1-year old to 2-year old is 0.91
- The fertility rate of 0-year olds is 0.1
- The fertility rate of 1-year olds is 1.2
- The fertility rate of 2-year olds is 0

Set up a recurrence, in human language, for $n_0(t)$, the number of 0-year-olds at time t. Also express it in our Maple notation, as a list `REC`.

Sol. to 1 Let $n_0(t), n_1(t), n_2(t)$ be the number of 0-year olds, 1-year olds, and 2-year olds respectively.

Using the data we

$$n_1(t) = 0.95 \cdot n_0(t-1)$$

$$n_2(t) = 0.91 \cdot n_1(t-1)$$

For future reference, let's express $n_2(t)$ in terms of $n_0(t)$. We have, since $n_1(t) = 0.95 \cdot n_0(t-1)$, replacing t by $t-1$ we get $n_1(t-1) = 0.95 \cdot n_0(t-2)$, so we have

$$n_2(t) = 0.91 \cdot 0.95 \cdot n_0(t-2)$$

Regarding fertility, we have

$$n_0(t) = 0.1 \cdot n_0(t-1) + 1.2 \cdot n_1(t-1) + 0 \cdot n_2(t-2)$$

So, in terms of $n_0(t)$ only

$$n_0(t) = 0.1 \cdot n_0(t-1) + 1.2 \cdot 0.95 \cdot n_0(t-2)$$

So the recurrence in **humanese** is

$$n_0(t) = 0.1 \cdot n_0(t-1) + (1.14) \cdot n_0(t-2)$$

and in our Maple notation

```
REC=[ 0.1, 1.14]
```

Now, just for fun, let's set the **Leslie matrix**. It is

$$L = \begin{bmatrix} 0.1 & 1.2 & 0 \\ 0.95 & 0 & 0 \\ 0 & 0.91 & 0 \end{bmatrix}$$

You are welcome to check that if you type in `M5.txt`

```
GrowthCe([ 0.1, 1.14]);
```

you would get the same answer as

```
Eigenvalues(Matrix([[0.1,1.2,0],[0.95,0,0],[0,0.91,0]]))[1];
```

(ignore the `+ 0I`).