Homework for Lecture 9 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file or .txt file) to

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by 8:00pm Monday, Oct. 4,, 2021.

Subject: hw9

with an attachment hw9FirstLast.pdf and/or hw9FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1. Use BOTH procedure `Orb` and **SPF**

`https://sites.math.rutgers.edu/~zeilberg/Bio21/M9.txt`

To find the stable fixed points of the following non-linear recurrencnes

$$x_n = f(x_{n-1})$$

for the following $f(x)$

(i) $2x(1-x)$, (ii) $(2.5)x(1-x)$, (iii) $(3,1)x(1-x)$, (iv) $\frac{4+x}{3+x}$, (v) $\frac{3+x}{4+x}$, (vi) $\frac{3+x+x^2}{4+x+2x^2}$
.

2. Consider the discrete dynamical system $x \to \frac{x+a}{x+b}$, where a and b are positive numbers.

Note that `FP` and `SFP` will **not** work if a and b are left as symbols. You are welcome to use the `solve` command to find explicit expressions, in terms of a and b of the two fixed points.

Then you are welcome to use `diff` command to find an expression C(a,b), such that the following is true

$x \to \frac{x+a}{x+b}$, where a and b are positive numbers, has a stable fixed point if and only if

$$-1 < C(a,b) < 1 \quad .$$

Then experiment with $a = 1, b = 2$, $a = 2, b = 3$, and $a = 12, b = 17$ to see whether the condition is met, and check your answers against the outputs of `Orb`, `FP` and `SFP` for these numerical values of a and b.

3. For an **arbitrary** k (between 1 and 4) find the equilibrium points of

$$x \to k\,x\,(1-x) \quad .$$

Prove that $x = 0$ is never stable, but that the other one is sometimes stable. What values of k is that other point fixed point stable? Use this to find the first **bifurcation point** when it switches from very one ultimate population value to going back-and-forth between two population values.

4. For an **arbitrary** k (between 1 and 4) find the equilibrium points of $x \to f(f(x))$

$$f(x) = kx(1-x) \quad .$$

you are welcome to use the Maple `solve` command to get explicit expressions in k for these four fixed points. Two of them are obviously the same as the fixed points of $x \to kx(1-x)$, and you already know that they are not stable.

Find a condition in k for the two new fixed points (of $x \to f(f(x))$ to be stable fixed points.

Use it to find **exactly** the second bifurcation points, when the population stops converging to a period 2 orbit, and starts going into a period 4 orbit.

Verify this theoretical prediction using `Orb` with $x_0 = 0.5$.

5. Read and understand (as much as possible) Mitchell Feigenbaum's seminal article:

`https://sites.math.rutgers.edu/~zeilberg/Bio21/MF78.pdf` .