

Homework for Lecture 24 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

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by 8:00pm Monday, Nov. 29,, 2021.

Subject: hw24

with an attachment hw24FirstLast.pdf and/or hw24FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

Comment: The goal of this homework and all the remaining ones is to prepare you to the 'basic concepts/skills' qualifying exam on Dec. 7. As I hope you know, this is a necessary condition for passing.

1. Carefully read, and understand, the posted solution to the attendance quiz for Lecture 24

<https://sites.math.rutgers.edu/~zeilberg/Bio21/att24S.pdf> .

If you get any of it wrong, explain what you did wrong.

2. For a certain particle, moving in a straight line, the rate of change of the rate of change of the rate of change of the acceleration is 120 meters/sec^3 . At the very beginning its velocity, acceleration, rate of change of acceleration, and rate of change of rate of change of the acceleration are 0 meters/sec , 0 meters/sec^2 , 0 meters/sec^3 , and 0 meters/sec^4 , respectively. How far from the starting point is the particle after 2 seconds?

3. (i) A ball is dropped vertically from a building of height 100 meters. How long will it take until it hits the ground. Ignore air resistance

(ii) A ball is dropped vertically from a building of height 100 meters. The air-resistance is twice times the mass times the velocity. Set up a differential equation with the appropriate initial conditions, and then use Maple's `dsolve` to find how long it will take to hit the ground.

4. Define

(a) A discrete-time dynamical system with one-quantity (variable). Write down its **format**.

(b) The orbit of a discrete-time dynamical system with one-quantity (variable) starting at $x(n) = x_0$ up to $n = K$.

(c) The **notion** of an equilibrium solution (Hint: it is a solution of the difference equation that is always constant)

(c) The **notion** of a stable equilibrium solution (Hint: it is a solution of the difference equation that is always constant and if you start $x(0)$ not too far from it, you also wind-up, in the long run very close to it.)

5.

(a) Describe how to **numerically** locate (at least approximately, but very reliably) the stable fixed points (aka stable discrete-equilibria), using orbits (implemented by `Orb`)

(b) In terms of the **underlying function**, call it $f(x)$ describe **how** to use algebra to find the set of **fixed points** (aka discrete-equilibrium-solutions)

(c) In terms of the **underlying function**, call it $f(x)$ describe how to use calculus to find the subset of the above set, the set of **stable fixed points** (aka discrete-stable equilibrium-solutions)

(d) Apply (a) (using `Orb` in `DMB.txt` with the appropriate inputs) , (b), (c) to the following discrete-time first-order dynamical system

(i)

$$x(n) = \frac{x(n-1) + 1}{x(n-1) + 2} \quad ,$$

(ii)

$$x(n) = \frac{5}{2}x(n-1)(1 - x(n-1)) \quad ,$$

(iii)

$$x(n) = \frac{7}{2}x(n-1)(1 - x(n-1)) \quad ,$$