

Homework for Lecture 22 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

ShaloshBEkhad@gmail.com

by 8:00pm Monday, Nov. 22,, 2021.

Subject: hw22

with an attachment hw22FirstLast.pdf and/or hw22FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1. Carefully read, and understand, the posted solution to the attendance quiz for Lecture 22

<https://sites.math.rutgers.edu/~zeilberg/Bio21/att22S.pdf> .

For each of the problems that you got wrong, explain what you did wrong, or what you were confused about. For example, if you tried to set-up a difference equation, write

'I did not realize that it is was continuous time'

OR:

'I have trouble with 'word problems' and had no clue how to translate from verbal English to mathematics'.

Then for each of the problems that you got wrong, make up two similar ones and solve them.

2. Set up a system of difference or differential equation (whatever is appropriate) for the following problems. Then use either `Orb` (or matrix algebra) or `dsolve`, respectively, to answer the question.

a There are two species of animals hares and lynxes. It turns out that

- The number of lynxes at the start of each year equals twice the number of lynxes at the start of last year plus three times the number of hares at the start of last year.
- The number of hares at the start of each year equals three times the number of lynxes at the start of last year plus the number of hares at the start of last year.

At the start of the first year there were 10 hares and 20 lynxes. How many hares and how many lynxes were there at the start of the tenth year?

b There are two species of animals hares and lynxes. It turns out that

- The rate of change (where the unit of time is a year) of the number of lynxes at any time equals

twice the number of lynxes plus three times the number of hares at that very same time.

- The rate of change of the number of hares at any time equals three times the number of lynxes plus the number of hares at that very same time.

At the very start ($t = 0$) there were 10 hares and 20 lynxes. How many hares and how many lynxes are there after ten years?

3. Carefully read

<http://sites.math.rutgers.edu/~zeilberg/Bio21/LadasSri.pdf>

and confirm the claims for some randomly chosen values of the parameters for Conjecture 1.

4. Carefully read

<http://sites.math.rutgers.edu/~zeilberg/Bio21/LadasSri.pdf>

and confirm the claims for some randomly chosen values of the parameters for Conjecture 2.

5. Find the fixed points and stable fixed points of the first-order difference equation.

$$x(n) = \frac{x(n-1)}{10 + x(n-1)} .$$

Where $x(0)$ can be anything **except** -10 .

6. (Optional, 30 brownie points). Prove (rigorously) that the stable fixed point that you found is a **global attractor**, i.e. wherever you start (except of course at $x(0) = -10$, you will, in the long-run wind-up in that stable fixed point.

Added Nov. 23, 2021: Congratulations to Anne Somalwar for solving it! See

<https://sites.math.rutgers.edu/~zeilberg/Bio21/HW22posted/hw22AnneSomalwarExtraCredit.pdf>