

Homework for Lecture 20 of Dr. Z.'s Dynamical Models in Biology class

Version of Nov. 12, 2021, thanks to Julian Herman, who spotted a typo, and won 5 dollars)

Email the answers (either as .pdf file and/or .txt file) to

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by 8:00pm Monday, Nov. 15,, 2021.

Subject: hw20

with an attachment hw20FirstLast.pdf and/or hw20FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1. Carefully read, and understand, the Maple code for the following procedures (type Help(ProcedureName); for instructions)

in the newly created Maple package

<https://sites.math.rutgers.edu/~zeilberg/Bio21/DMB.txt> ,

- SIRS
- EquP
- SEquP
- TimeSeries
- PhaseDiag

Use them to compute (i) the equilibrium points (ii) the stable equilibrium points, and plot the evolution in the SIRS model that has total population of 1000 people that at the start had 800 susceptible (but not infected) and 200 infected (and hence at the start no one removed yet), of the number of Susceptible individuals as a function of time from $t = 0$ to $t = 10$, and another plot for the number of infected individuals, and finally a phase-diagram that ignores time, but shows how the pair (Susceptible, Infected) evolves from $t = 0$ to $t = 10$, using mesh size 0.01 for the discretization, for the following values of β (recall that $N = 1000$)

For $\beta = 0.3 \cdot \frac{\nu}{1000}$, $\beta = 0.9 \cdot \frac{\nu}{1000}$, $\beta = 3.9 \cdot \frac{\nu}{1000}$

for each of the following choices

(i) $\nu = 2$, $\gamma = 5$

(ii) $\nu = 3, \gamma = 6$

(iii) $\nu = 4, \gamma = 1$

(iv) $\nu = 7, \gamma = 10$

Confirm that the horizontal asymptotes of the time series for the number of susceptible, and the number of infected persons are consistent with the stable equilibrium points.

(Note that when Maple draws a graph the horizontal axis is usually not $y = 0$, so don't be fooled)

2. Using `RandNice([x,y],3)` 3 times, (call it `F`) generate three random transformations from R^2 to R^2 .

For each of them do the following

(i): Find the set of equilibrium points

(ii): Find the set of stable equilibrium points

If the set of equilibrium points is empty, try again, until you get a transformation that has at least one equilibrium point.

Then do the following

(iii): Plot the Time series for $x(t)$ and $y(t)$ with mesh-size 0.01 from $t = 0$ to $t = 10$, as well as phase diagrams in the (x, y) -plane for each case. Taking initial conditions close (but not too close) to the stable equilibrium points. Confirm that the horizontal asymptotes coincide with those predicted by the stable equilibria. Also draw the phase diagrams.

3. Carefully read, and understand, the Maple code for `Orbk`, and use it to find, numerically, the stable equilibrium point of the difference equation

$$x_{n+1} = \frac{3 + x_{n-1} + x_{n-2} + x_{n-3}}{1 + x_n + x_{n-2}}$$

(experiment with different initial conditions and see if you get the same long-term behavior). Then confirm it using `ToSys` followed by `SFP`.