Homework for Lecture 18 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

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by 8:00pm Monday, Nov. 8,, 2021.

Subject: hw18

with an attachment hw17FirstLast.pdf and/or hw17FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1.

Carefully read the answer to the first question in

`https://sites.math.rutgers.edu/~zeilberg/Bio21/att18S.pdf` .

understand it and then write a Maple program that inputs numbers a, b, c, d, e, call it, $C(a, b, c, d, e)$ and outputs the answer to the following question

a chickens lay b eggs in c days, how many eggs do d chickens lay in e days?

Check that $C(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 3, 3) = 6$ (as in the original problem).

2.

Carefully read the answer to the second question in

`https://sites.math.rutgers.edu/~zeilberg/Bio21/att18S.pdf` .

understand it and then write a Maple program that inputs numbers a, b, k, call it, $W(a, b, k)$ and outputs the answer to the following question

- A and B can fill a cistern in a hours,

- A and C can fill the same cistern in b hours,

- B can fill k-times as fast as C

Find how long C would take to fill the cistern, working alone.

Check that $W(4, 5, 2) = 20$.

3. Recall that in order to find all the equilibrium points of a continuous-time (first-order) system

$$\mathbf{x}'(t) = F(\mathbf{x}(t)) \quad ,$$

where F is a transformation from R^k to R^k you do the following

• (i) Use algebra to solve $F(\mathbf{x}) = \mathbf{0}$, getting a (usually) finite set of points in R^k. These are **all** the equilibrium points (that live in R^k) of the dynamical system (but so far you don't know whether they are stable or not).

• (ii) Find the **Jacobian matrix**, $\mathbf{J}(\mathbf{x})$ in general, featuring $x_1, \dots, x_k$.

• (iii) For each of the points that you found in the algebraic step (i), plug it in the general Jacobian matrix of (ii), getting a certain **numerical** $k \times k$ matrix. Ask Maple to find its **eigenvalues**. If all of them have **negative real part** then the examined equilibrium point is **stable**, otherwise not. (In the borderline case of purely imaginary eigenvalue (or 0) it is sometimes called **semi-stable**.)

Use the above to find all the equilibrium points of the 2-dimensional continuous dynamical system

$$x'(t) = x(t)(1 - x(t) - y(t)) \quad ,$$

$$y'(t) = x(t)(3 - 2x(t) - y(t)) \quad .$$

Then find out which ones of them are stable and which ones are not.

4. Using procedure `Dis2(F,x,y,pt,h,A)`, with $h = 0.01$, and $A = 10$

`https://sites.math.rutgers.edu/~zeilberg/Bio21/M18.pdf` .

confirm numerically the answers of problem **3.**. Take `pt` to be close (but not the same) as the above-mentioned equilibrium points, (for example, if the given equilibrium point is $[a, b]$ take $pt = [a + 0.1, b + 0.1]$), and see whether it tends to $[a, b]$ or rather escapes away.

5. Read section 6.6. of Leah Edelstein-Keshet's book

`https://sites.math.rutgers.edu/~zeilberg/Bio21/keshet/keshet6.pdf`

and convince yourself that the SIRS dynamical system given in Eqs (28) (once we use $R = N - S - I$) is represented by procedure

`SIRS(s,i,beta,gamma,nu,N)` ,

i.e. it gives the **underlying transformation** where we used s and i rather than S and I (in Maple $I = \sqrt{-1}$).

Using procedure

`Dis2(SIRS(s,i,beta,gamma,nu,N),x,y,[N-30,30],0.01,10)` in

`https://sites.math.rutgers.edu/~zeilberg/Bio21/M18.pdf` .

with $\beta = 0.01$, $\nu = 1$ (so $\frac{\nu}{\beta} = 100$)

with $N = 50$, $N = 80$, $N = 120$, confirm the prediction that the epidemic will be eradicated (eventually the number of infected individuals will go to 0) if $N < \frac{\nu}{\beta} = 100$, but will persist if $N > \frac{\nu}{\beta} = 100$. If $N = 120$, in the long-run, how many individuals will be infected?

Comment $\frac{N\beta}{\nu}$ is the famous R_0-factor, aka **infectuous contact number**.