Homework for Lecture 16 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

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by 8:00pm Monday, Nov. 1,, 2021.

Subject: hw16

with an attachment hw16FirstLast.pdf and/or hw16FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

0. Pick or suggest a final project. You are welcome to pick your own team-mates (max. size: 3 students including the leader) by Nov. 1, 2021. Otherwise I will assign you a project and form random teams.

1. (For everyone!, some people got close, but no one did it fully)

Carefully read,

`https://sites.math.rutgers.edu/~zeilberg/Bio21/att16.pdf` .

understand it. Then do the similar questions (with all the parts) for the following difference equation

(a)

$$x(n) = x(n-1)(\frac{5}{3} - x(n-2)) \quad .$$

(b)

$$x(n) = x(n-1)(2 - x(n-2)) \quad .$$

Confirm your results **numerically** by running

`Orbk(k,z,f,INI,K1,K2)`

with $K1 = 1000$ and $K2 = 1020$.

(in the Maple code given here:

`https://sites.math.rutgers.edu/~zeilberg/Bio21/M15.txt` .

2. (A little bit challenging, but do your best, it is mandatory to attempt it). Consider the **family** of second-order difference equations, featuring the **parameter** a, assumed to be positive.

$$x(n) = x(n-1)(a - x(n-2)) \quad .$$

Find all the equilibrium points (0 is always one of them, but find the other one), and find conditions on a for the stability of $x = 0$ and conditions for a for the stability of the other equilibrium point.

3. (Optional challenge, 20 brownie-points). Consider the **family** of third-order difference equations, featuring the **parameters** a, b, assumed to be positive.

$$x(n) = x(n-1)(a - x(n-2))(b - x(n-3)) \quad .$$

Find all the equilibrium points (0 is always one of them, but find the other ones, expressed in terms of a and b), and find conditions on a, b for the stability of $x = 0$ and conditions on (a, b) for the stability of the other equilibrium points.

Confession: I did not do it myself, and I have no idea how hard it is.

4. For each of the following (autonomous) first-order ordinary differential equations

(i) Find all the equilibrium points

(ii) For each of them decide whether it is a stable equilibrium or not

(iii) Confirm then by using

```
plot(Dis1(F,y,y0,h,A))
```

with $h = 0.01$ and $A = 20$.

(it is available from

`https://sites.math.rutgers.edu/~zeilberg/Bio21/M15.txt`

)

with $h = 0.01$

Worked Example: $x'(t) = x(t)(3 - x(t))$

Unerlying function : $F(x) = x(3 - x)$ (recall that the **format** of a first-order autonomous differential equation is $x'(t) = F(x(t))$)

(i): Equilibrium points: Solve $x(3 - x) = 0$, getting $x = 0$ and $x = 3$.

(ii): $F'(x) = 3 - 2x$.

When $x = 0$, $F'(0) = 3$, this is **not** negative, so it is **unstable**

When $x = 3$, $F'(0) = -3$, this is **negative**, so it is **stable**.

(iii)

To invesigate numerically the status of $x = 0$ you do:

plot(Dis1(y*(3-y),y,0.01,0.01,20));

Note: for this simple case, Maple can solve it, so you can also use

```
plot(op(2,dsolve(diff(x(t),t)=x(t)*(3-x(t)), x(0)=0.01,x(t))),t=0..10);
```

(but for the more complicated cases coming up,`dsolve` is too slow and unreliable).

The horizontal asymptote is the **other equilibrium point** $x = 3$! So $x = 0$ is indeed unstable.

To invesigate numerically the status of $x = 3$ you do:

plot(Dis1(y*(3-y),y,0.01,3.01,20));

(OR `plot(op(2,dsolve(diff(x(t),t)=x(t)*(3-x(t)), x(0)=3.01,x(t))),t=0..10);`)

a: $x'(t) = x(t)(3 - x(t))(5 - x(t))$

b: $x'(t) = x(t)^2\,(3 - x(t))(5 - x(t))(7 - x(t))$.

(You are welcome to use Maple for differentianting $F(x)$ and for plugging-in).