Homework for Lecture 15 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file and/or .txt file) to

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by 8:00pm Monday, Oct. 25,, 2021.

Subject: hw15

with an attachment hw15FirstLast.pdf and/or hw15FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1. Read and understand the Maple code for procedure Dis1(F,y,y0,h,A)

`https://sites.math.rutgers.edu/~zeilberg/Bio21/M15.txt` .

2.

(i) Let a_i := the i-th digit of your RUID (if it is 0 make it 2).

Using Maple's `dsolve` solve the following initial value problems

$$x'(t) = (a_3 - x(t))(a_4 - x(t))(a_7 - x(t)) \quad , x(0) = \frac{a_3 + a_4}{2} \quad ,$$

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then use Maple's **plot** command to plot these solutions from $t = 0$ to $t = 5$.

(ii) Use `Dis1`, followed by Maple's built-in command `plot`, to find the results of **discretization**, first with $h = 0.1$ and then with $h = 0.01$. Convince yourself that they look similar.

Hint: The syntax for plotting a function f(t)=Expression(t)

plot(Expression, t=a..b).

For example to plot the function $x(t) = \sin(t)$ from $t = 0$ to $t = 10$ you type

`plot(sin(t),t=0..10);`

To plot a list of points `L` (the output of `Dis1`) you use the same Maple command `plot` but the syntax is

`plot(L);`

2.

(i) Fully by hand convert the **fourth-order** recurrence

$$x(n) = \frac{x(n-1) + 2x(n-2) + 3x(n-3) + 11x(n-4)}{x(n-1) + x(n-3)} \qquad x(0) = 1\,,\, x(1) = 5\,,\, x(2) = 5\,,\, x(3) = 2$$

into a **first-order** system with four sequences $x_1(n), x_2(n), x_3(n), x_4(n)$, where you renamed $x(n)$, $x_1(n)$.

(ii) Check that this agrees with the output of `ToSys(k,z,f,INI)`. What do you enter for `k`, `f`, and `INI`? What is the output?

3. Use procedure `Orbk` (see `Help13();`) to numerically find a stable fixed points (if it exists) of the second-order recurrence (with the given initial conditions)

$$x(n) = (1 - x(n-1))\,(1 - x(n-2)) \quad , \quad x(0) = 2.5 \quad , \quad x(1) = 2.7 \quad .$$

Confirm this rigorously by first using `ToSys` and then `SFP2`.

4. Carefully read section 3.6 of Leah Edelstein-Keshet book

`https://sites.math.rutgers.edu/~zeilberg/Bio21/keshet/keshet3.pdf`

Fill-in all the "blanks" and fully derive the Hardy-Weinberg "dynamical system". But instead of lumping-up, for example "Father=AA and Mother=Aa" and "Mother=AA and Father=Aa" into one-category, keep the 9 categories separate.

You can make a 3×3 table, and in each of the boxes sub-divide it into three compartments

AA, Aa, aa

and exrpess then in terms of u, v, w (of course sometimes you would get 0, for example if both parents are AA, then the number of aA as well as the number of aa is obviously 0).

By adding-up all the terms for AA, Aa, aa, each derive Eqs. ((53a), (53b), (53(c)) that formed the basis of `HW3(u,v,w)` and `HW2(u,v)`.

5. (Optional Challenge, 20 brownie points) Generalize `HW3(u,v,w)` to write a procedure

`HW3g(u,v,w,M)`

where (let's number the types AA,Aa,aa, by $1, 2, 3$) M is a 3×3 matrix (given as a list of lists) such that

$M[i][j]$ is the fraction of an off-springs that a type-i father and type j mother survive to the next generation.

Then use it to write

```
HW2g(u,v,M)
```

and use `Orb2` to see whether the persistence of genes still holds. (Note: I did not do it myself, so I don't know the answer, and would like to know).

The case of $M = [[1,1,1],[1,1,1],[1,1,1]]$ corresponds to the original case.

Hint: (i) You should adapt the table you did in **4** but now you would get M_{ij} all-over the place.

(ii) **Very important**: Before the sum of all the contributions from the $9 \times 3 = 27$ entries in the table was automatically 1. This is no longer true now, so at the end of the day you have to **normalize** the new u, v, w by dividing by this (complicated!) sum.