Homework for Lecture 14 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file or .txt file) to

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by 8:00pm Monday, Oct. 25,, 2021.

Subject: hw14

with an attachment hw14FirstLast.pdf and/or hw14FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1. (Mandatory for all students who did not get today's "coming on time" attendance quiz completely right).

Carefully read,

`https://sites.math.rutgers.edu/~zeilberg/Bio21/att14.pdf` .

understand it. Then do the similar quesion for the following dynmatical system

$$x(n) = x(n-1) + y(n-1)^4 - \frac{1}{16} \quad ,$$

$$y(n) = x(n-1)^2 + y(n-1) - \frac{1}{9} \quad .$$

but this time do it for **all** the fixed points.

2. Draw the **directed graph** with 16 vetrices, labeled $1, \dots, 16$ of the Dynamical system

$$x \rightarrow x^3 \quad mod\ 17 \quad .$$

Find all the periodic cycles (including those of size 1, aka as *fixed points*), and write down explicitly all the 16 trajectories for each of the possible starting points. For example, the one that starts with 1 is simply

$$[1, 1]$$

The one that starts with 2 is

$$[2, 8, 2]$$

3. Let $T_k(n)$ be the following finite Dynamical system that takes place in all k-digit positive integers.

- Arrange the digits in decreasing order, call it $L(n)$
- Arrange the digits in incrasing order, call it $S(n)$
- Define $T_k(n) = L(n) - S(n)$.

For example if $k = 3$ and $n = 327$, then $L(327) = 732$, $S(n) = 237$ and $T_3(327) = 732 - 237 = 495$

(i) Take 10 random 2-digit numbers, and , by hand, find the trajectories of $T_2(n)$, until the first repeat, and determine their ending cycle (that may or may not be of length 1, i.e. a fixed point).

(ii) Take 5 random 3-digit numbers, and , by hand, find the trajectories of $T_3(n)$, until the first repeat, and determine their ending cycle (that may or may not be of length 1, i.e. a fixed point).

(iii) Take 3 random 4-digit numbers, and , by hand, find the trajectories of $T_4(n)$, until the first repeat, and determine their ending cycle (that may or may not be of length 1, i.e. a fixed point).

4. Read and understand the procedures `RevOp(n,k)` and `RevOpTr(n,k)` in the Maple code

`https://sites.math.rutgers.edu/~zeilberg/Bio21/M14.txt` ,

and convince yourself that it implements $T_k(n)$. Use it to find all the *periodic orbits* (aka *cycles*) of $T_3(n)$ and $T_4(n)$. Are they all fixed points?

5. Consider the dynmailca system, defined on the **infinite** set of positive integers

$$n \rightarrow \frac{n}{2} \quad if \quad n \quad is \quad even \quad , \quad n \rightarrow \frac{3n+1}{2} \quad if \quad n \quad is \quad is \quad odd \quad .$$

Take five random integers and find their trajectories. What do they end up with?

6. (Optional, 10000 brownie points) Find all the periodic orbits of the dynamical system in **5**.