Homework for Lecture 12 of Dr. Z.'s Dynamical Models in Biology class

Email the answers (either as .pdf file or .txt file) to

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by 8:00pm Monday, Oct. 18,, 2021.

Subject: hw12

with an attachment hw12FirstLast.pdf and/or hw12FirstLast.txt

Also please indicate (EITHER way) whether it is OK to post

1. For each of the following two (single variable) functions find all the fixed points, and for each of them, decide whether they are stable fixed points.

(i)

$$x \to x^3 - 6x^2 + 12x - 6$$

(ii)

$$x \to x^4 - \frac{13\,x^2}{36} + x + \frac{1}{36}$$

2. Find the **linearizations** of the given functions at the designated points, and compare the exact value with the approximate values given by the linearization.

(i): $f(x, y) = \sqrt{x + 4y}$ at $(1, 2)$. The values at $(0.95, 1.02)$.

(ii): $f(x, y, z) = x^3 y^4 z^5$ at $(1, 1, 1)$. The values at $(1.01, 1.02, 0.99)$.

(iii): $f(x_1, x_2, x_3, x_4) = \sqrt{x_1 + x_2 + x_3 + x_4}$, at $(1, 1, 1, 1)$. The values at $(1.01, 1.01, 0.99, 0.99)$.

3. What is the **Jacobian matrix** (not to be confused with the Jacobian determinant) of the following transformation

$$(x, y) \to (\frac{x}{y+1}, \frac{y}{x+1}) \quad ,$$

at the point $(1, 1)$.

4. What is the **Jacobian matrix** (not to be confused with the Jacobian determinant) of the following transformation

$$(x, y, z) \to (x + y + z\,,\, x^2 + y^2 + z^2\,,\, x^3 + y^3 + z^3\,) \quad ,$$

at the point $(1, 1, 1)$.

5. In your words explain why it makes sense that a fixed point (x_0, y_0) of a transformation $(x, y) \to (f(x, y), g(x, y))$, i.e. a point in R^2 such that $x_0 = f(x_0, y_0)$ and $y_0 = g(x_0, y_0)$ is a **stable fixed point** if its **Jacobian matrix** , at that point, i.e.

the 2×2 **numerical matrix**

$$\begin{pmatrix} f_x(x_0, y_0) & f_y(x_0, y_0) \\ g_x(x_0, y_0) & g_y(x_0, y_0) \end{pmatrix} \quad ,$$

has all its **eigenvalues** with absolute value less than 1.