Take Home Final Exam for Dr. Z.'s Mathematical Models in Biology, Fall 2021

Please email `ShaloshBEkhad@gmail.com`, no later than **Tuesday, Dec. 14, 9:00am**, with

Subject: FINAL

and two attachments:

• `FirstLastFinal.txt`. Download the blank form

`https://sites.math.rutgers.edu/~zeilberg/Bio21/BlankAnswerKey.txt` ,

rename it `FirstLastFinal.txt` , fill-in the numerical answers, and email it bac)

• `FirstLastFinal.pdf`: Either hand-written or typed, explaining your answers, and giving (if applicable) the Maple syntax, and/or Maple code, that got your the answers.

This is "open book" and, open computer (of course, you should use Maple and the Maple package `DMB.txt`), and "open internet", but **no help from other people**. The exam will be followed up by a short oral exam (possibly for randomly selected subset) to make sure that you did it "all by yourself". During that oral exam I will also make sure that you mastered the basic concepts that you messed up with in the 'qualifying exam'.

Each question is worth 10 points. The maximum possible score is 100.

1. At day 0 there is one rabbit , at day 1 there is still one rabbit , at day 2 there are two rabbits. It so happens that the number of rabbits at any given day is twice the number of rabbits yesterday minus the number of rabbits three days ago. What is (in decimals) the number of rabbits at day 1000 divided by the number of rabbits at day 999?

2. A certain species with carrying capacity 1 has a **rate of change** that is equal to

"$\frac{5}{2}$ times its current quantity" times

"1 minus its current quantity" times

"1 minus one half of its current quantity".

(a) Find **all** the **equilibrium solutions**.

(b) Find **all** its **stable equilibrium solutions** (if there are none, say so) .

(c) If at time $t = 0$, its value is 0.1, what would its value be at $t = 100$?

3. A certain species with carrying capacity 1 is such that its **quantity today** is

" $\frac{5}{2}$ times its quantity yesterday" times

"1 minus its quantity yesterday" times

"1 minus one half of its quantity yesterday".

(a) Find **all** the **equilibrium solutions**.

(b) Find **all** the **stable equilibrium solutions** (if there are none, say so)

(c) If at day zero, its value is 0.1, what would its value (with ten decimal accuracy) be at day 1000?

4. At the first generations there are equal proportions of people with genotypes AA, Aa, and aa. Under the **Hardy-Weinberg** hypotheses (and Law)

(a) What proportion of the second generation would have genotype Aa?

(b) What proportion of the 1000^{th} generation would have genotype Aa?

5. At the first generations there are equal proportions of people with genotypes AA, Aa, and aa.

All three female genotypes are equally likely to mate with the three male genotypes, **except** that an AA female is twice as likely to mate with an Aa male than all the other eight mating combinations.

(a) What proportion of the second generation would have genotype Aa?

(b) What proportion of the 1000^{th} generation would have genotype Aa?

6. Suppose that

$$x(n) = \frac{1 + x(n-1) + y(n-1)}{2 + x(n-1) + 3y(n-1)} \quad , \quad y(n) = \frac{1 + x(n-1) + 3y(n-1)}{3 + x(n-1) + 2y(n-1)} \quad .$$

If $x(0) = 100$, and $y(0) = 1000$, what do you think that

$y(100000000000000000000000000000000000000000000000000000)$

would be equal to (rounded to ten decimal points)?

7. (i) In the SIRS model (Edelstein-Keshet, section 6.6),

`https://sites.math.rutgers.edu/~zeilberg/Bio21/keshet/keshet6.pdf` ,

with a population of $N = 1000$, and the values of the parameters γ amd ν being, respectively, $\gamma = 0.5$ and $\nu = 100$, if at the start there are 300 susceptible, 300 infected, (and hence 400 removed)

(a) If $\beta = 0.05$, in the long-run how many removed individiuals would there be?

(b) If $\beta = 1.4$, in the long-run how many removed individuals would there be?

(c) What value of β would be the **cut-off** when, in the long-run, there would start to be a non-zero number of infected people?

8. In the protein model given in Eq. [4.1] of Ellner-Guckenheimer

`https://sites.math.rutgers.edu/~zeilberg/Bio21/dmb/dmb4.pdf` ,

Figure 4.3 (p. 115, p. 9 of the pdf) shows a plot of the evolution of the six proteins, where the parameters are $\alpha_0 = 0$, $\alpha = 1$, $\beta = 0.2$, and $n = 2$. The initial conditions are also given there (but they are not important for the long-term behavior). There is a clear (common) horizontal asymptote, at a certain height above the t-axis.

(a): Using the Maple package `DMB.txt` (in particular `GeneNet` and `SEquP`), find, to an accuracy of 10 decimals, the exact height of that horizontal asymptote, in other words the **stable equilibrium** (that happens to be the same for all six proteins).

(b): With the same initial conditions, and parameters, **except** that instead of $\alpha = 1$ you take $\alpha = 3$, do you still have a horizontal asymptote? What is the height of the horizontal asymptote, above the t-axis, now?

(c) With the same initial conditions, and parameters, **except** that instead of $\alpha = 1$ you take $\alpha = 50$, Fig. 4.2 of that book clearly shows oscillatory behavior, indicating that there is no stable equilibrium. Find a value of α (with all the other parameters exactly the same) such that for that value of α there still is a stable equilibrium (but it takes a very long time to reach it!), but for $\alpha + 0.01$ there is no longer a stable equilibrium.

9. In the Chemostat model (Edelstein-Keshet, section 4.5),

`https://sites.math.rutgers.edu/~zeilberg/Bio21/keshet/keshet4.pdf` ,

Equations (19a) and (19b), with parameters $\alpha_1 = 2.5$ and $\alpha_2 = 2.7$,

(a): What would be the value of the Bacterial population density after a very long time?

(b): What would be the value of the Nutrient concentration after a very long time?

10. Consider a mini-internet with nine web-pages, $1, 2, \ldots 9$.

• A random surfer who is currently at either web-page 1, 2, or 3, will, at the next time-step, stay at the same page with probability 0.2 but if she leaves that page, she is equally likely to go to one of the other eight pages.

• A random surfer who is currently at either web-page 4, 5, or 6, will, at the next time-step, stay at the same page with probability 0.4, but if she leaves that page, she is equally likely to go to one of the other eight pages.

• A random surfer who is currently at either web-page 7, 8, or 9, will stay, at the next time-step, at the same page with probability 0.6 but if she leaves that page, she is equally likely to go to one of the other eight pages.

In the **long-run**

(a): what is the probability that a random surfer will be at web-page 1?

(b): what is the probability that a random surfer will be at web-page 9?