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Math 336

Project 4

Study numerically the ultimate periodic orbits for various parameters of the generalized discrete Logistic Equation

x(n)=k x(n-1)(1-x(n-1)).

These generalizations should have more parameters. For example:

x(n)=k x(n-1)^a (1-x(n-1))^b ,

that has three parameters (k, a, and b). Also study second- (and higher-) order difference equations, e.g.

#x(n)=k x(n-1)(1-x(n-1))(1-x(n-2)) ,

and more generally

x(n)=k x(n-1)^a(1-x(n-1))^b(1-x(n-2))^c ,

etc.

I studied numerically the activity of variations of the logistic equation to try and see the activity of the orbits. What I first tried to look at was the original logistic equation,

```
x(n)=k*x(n-1)*(1-x(n-1)
```

and test to see the activity it presents with negative values of k. The equation is originally used as a way to model population growth, with k being a fertility rate of the population. In that context, it makes no sense to have a negative fertility rate, but I am only trying to test the activity of the equation, so constraints are thrown out the window for my numerical analysis. The only constraints I followed were that my values of x for my initial conditions were always between 0 and 1, as they represent population size and since I am only testing the different values of parameters so the values of x for initial conditions are of no concern to me.

Onto the analysis of a negative k, I first wanted to see what similarities it shared with the original equation. I started testing with k values between -1<k<0, and I noticed that the behavior was the same as the positive counterparts. Eventually the orbit reached a stable fixed point of 0.

```
Orb([-.1*x*(1-x)],[x],[.1],1000,1010)
9.073501848 10^(-1002)], [-9.073501848 10^(-1003)], [9.073501848 10^(-1004)], [-9.073501848 10^(-1005)], [9.073501848 10^(-1006)], [-9.073501848 10^(-1007)], [9.073501848 10^(-1008)],
  [-9.073501848 10^(-1009)], [9.073501848 10^(-1010)], [-9.073501848 10^(-1011)], [9.073501848 10^(-1012)]]
```

```
Orb([-.9*x*(1-x)],[x],[.1],1000,1010)
[1.584710705 10^(-47)], [-1.426239634 10^(-47)], [1.283615671 10^(-47)], [-1.155254104 10^(-47)], [1.039728694 10^(-47)], [-9.357558246 10^(-48)], [8.421802421 10^(-48)], [-7.579622179 10^(-48)],
  [6.821659961 10^(-48)], [-6.139493965 10^(-48)], [5.525544568 10^(-48)]]
```

You'll notice that as we get to more negative values it appears our orbit, as k gets more negative, is looking as if it'll stop tending towards 0 if we keep decreasing the value of k. We turn out to be correct, because as soon as our value of k hits -1 we lose our stability at 0.

```
> Orb([-1*x*(1-x)],[x],[.1],1000,1010)
[[0.02196474294], [-0.02148229301], [0.02194378192], [-0.02146225236], [0.02192288063], [-0.02144226794], [0.02190203880], [-0.02142233950], [0.02188125614], [-0.02140246677],
    [0.02186053236]]
```

This is the same from what we would see in the case of a positive 1 value, with a k value of 1 we also wouldn't reach a stable fixed point. The difference here is that soon after, once we get into more negative values, we do begin to see a periodic orbit. This is the behavior we're really interested in.

```
> Orb([-1.1*x*(1-x)],[x],[.1],10000,10010)
[[0.3365056467], [-0.2455965560], [0.3365056467], [-0.2455965560], [0.3365056467], [-0.2455965560], [0.3365056467], [-0.2455965560], [0.3365056467], [-0.2455965560],
    [0.3365056467]]
> Orb([-1.2*x*(1-x)],[x],[.1],1000,1010)
[[0.4652146415], [-0.2985479746], [0.4652146415], [-0.2985479746], [0.4652146415], [-0.2985479746], [0.4652146415], [-0.2985479746], [0.4652146415], [-0.2985479746],
    [0.4652146415]]
> Orb([-1.3*x*(1-x)],[x],[.1],1000,1010)
[[0.5522237189], [-0.3214544882], [0.5522237189], [-0.3214544882], [0.5522237189], [-0.3214544882], [0.5522237189], [-0.3214544882], [0.5522237189], [-0.3214544882],
    [0.5522237189]]
```

The period of orbit 2 is seen between -1.4<k≤-1.1. Since -1.0 did not show a periodic orbit of 2, nor did it show stability, the bifurcation value for k must be somewhere between -1.1 and -1.0. Through rigorous testing with the Orb function from the DMB.txt I was able to find an approximate value for that bifurcation point. Our first bifurcation value into a period 2 orbit

begins at approximately k=-1.00792. The values before it indicated that we were eventually heading into that direction, but only at -1.00792 did it become obvious after 1010 timesteps.

```
> Orb([-1.00792*x*(1-x)],[x],[.1],1000,1010)
[[0.09231133824], [−0.08445357152], [0.09231133824], [−0.08445357152], [0.09231133824], [−0.08445357152], [0.09231133824], [−0.08445357152], [0.09231133824], [−0.08445357152],
  [0.09231133824]]
```

If we were to find anything of interest to us next it'd be to see if the periodic orbit doubles as it did in the original case with positive values of k, and so to find out I kept running the Orb function with values of k that kept going down by increments of .1. Eventually reaching -1.4 you notice the change in behavior immediately.

```
> Orb([-1.4*x*(1-x)],[x],[.1],1000,1010)
[[0.6166606851], [−0.3309463984], [0.6166606837], [−0.3309463989], [0.6166606851], [−0.3309463984], [0.6166606837], [−0.3309463989], [0.6166606851], [−0.3309463984],
  [0.6166606837]]
```

A period of 4 orbit is shown by the Orb function. That indicated that between -1.4 & -1.3 we had a period doubling occurring, another bifurcation value. Again, through rigorous testing I had to approximate where that point was. This bifurcation value ended up being at -1.31960.

```
> Orb([-1.31960*x*(1-x)],[x],[.1],1000,1010)
[[0.5662949376], [−0.3241003332], [0.5662949375], [−0.3241003333], [0.5662949376], [−0.3241003332], [0.5662949375], [−0.3241003333], [0.5662949376], [−0.3241003332],
  [0.5662949375]]
```

Could argue that the numbers are close enough that maybe the difference in the last place of the digits is due to a Maple rounding error, but every point prior to this clearly showed a periodic orbit of 2, and so the first point which was showing a movement away from that behavior is what is approximated to be the value where the bifurcation starts to occur. Every point after this, meaning, <-1.31960, started to clearly exhibit a period 4 orbit.

With the negative values, it became something to note that these k values went into periodic orbits much quicker than how long it took for the positive values of k to exhibit them. In that I mean, the absolute value of these k-values is less than the absolute value of the k-values of the original equation where such an event was occurring. It wasn't until somewhere around 3.1 where we saw our first bifurcation value, in the original logistic case. Already over here we've hit a period of 4 already at -1.31960. Soon enough we were going to hit the period of 8, 16, and then eventually chaos.

It did not take long for this period of 4 to eventually turn into a period of 8, we see the period 8 behavior begin at -1.6 at first, and then through further analysis we approximate its value between -1.6 & -1.5 which turned out to be at -1.52205.

```
> Orb([-1.52205*x*(1-x)],[x],[.1],1000,1030)
[[0.4696227664], [−0.3791079883], [0.7957747029], [−0.2473594947], [0.4696227656], [−0.3791079882], [0.7957747025], [−0.2473594951], [0.4696227664], [−0.3791079883],
 [0.7957747029], [−0.2473594947], [0.4696227656], [−0.3791079882], [0.7957747025], [−0.2473594951], [0.4696227664], [−0.3791079883], [0.7957747029], [−0.2473594947],
 [0.4696227656], [−0.3791079882], [0.7957747025], [−0.2473594951], [0.4696227664], [−0.3791079883], [0.7957747029], [−0.2473594947], [0.4696227656], [−0.3791079882],
 [0.7957747025]]
```

Quickly again we dissolve into a period of 16, not long after, at -1.56 we first notice it, and through rigorous testing to 5 decimal points again, I approximate that the period of 16 did indeed start at -1.56000.

```
> Orb([-1.56*x*(1-x)],[x],[.1],1000,1020)
[[0.5125741976], [−0.3897533478], [0.8449911912], [−0.2043304817], [0.3838870269], [−0.3689677329], [0.7879636766], [−0.2606399968], [0.5125741995], [−0.3897533476],
 [0.8449911907], [−0.2043304821], [0.3838870277], [−0.3689677332], [0.7879636772], [−0.2606399961], [0.5125741976], [−0.3897533478], [0.8449911912], [−0.2043304817],
 [0.3838870269]]
```

As we’ve noticed in the original discrete logistic equation with positive values of k, the space between bifurcation values quickly begins to decrease as we reach greater periodic orbits. That of course held true for this as well, and eventually the period doubling became a bit more difficult to catch, but eventually once I got past -2 for a value of k, Maple was no longer able to evaluate the decimal values of the orbit. For this, we say that the equation dissolved into chaos. Not unlike what we had seen in class, when we used k values greater than 4.

Though my project is mainly about the periodic orbits of variations to the logistic equation, with these negative k values I was curious to see if I’d find Feigenbaum’s constant to be present. I did work on that to find the ratio of the length between sequential bifurcation values.

$$\left[\frac{a(n-1) - a(n-2)}{a(n) - a(n-1)} \right]$$

Where a is the bifurcation value where a period doubling occurs. Could only test this out after we received our period of 8 bifurcation value. Through testing I did not see Feigenbaum's first constant, so I ended my search there as it would have got me nowhere.

I next tried a variation of the discrete logistic equation. I added a spare capacity of the population at x(n-2). The full equation is shown below.

$$x(n) = k\, x(n-1)\,(1 - x(n-1))\,(1 - x(n-2))$$

To work with this, I needed more than the Orb function as I now had a second order case. I now needed to convert this into a system, in order to work with it. As we did in class, we substituted these x(n)'s out and created new variables [z[1],…z[k]] for the k^{th} order difference equation. So in this case we only needed to substitute a z[1] and z[2] for x(n-1) and x(n-2) respectively. Our system became:

$$\left[k z_1 (1 - z_1)(1 - z_2), z_1\right]$$

Now I was free to use Orb to numerically analyze the behavior of this equation. I first began to analyze values of 0<k<1. Since the first order case, showed a stable fixed point of 0 at these points, I'd expect to see the same here. I was correct in my assumptions.

```
> F:=.1*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],1000,1010)
```

$$[[8.582526351\ 10^{-1002}, 8.582526351\ 10^{-1001}], [8.582526351\ 10^{-1003}, 8.582526351\ 10^{-1002}], [8.582526351\ 10^{-1004}, 8.582526351\ 10^{-1003}], [8.582526351\ 10^{-1005}, 8.582526351\ 10^{-1004}], [8.582526351\ 10^{-1006}, 8.582526351\ 10^{-1005}], [8.582526351\ 10^{-1007}, 8.582526351\ 10^{-1006}], [8.582526351\ 10^{-1008}, 8.582526351\ 10^{-1007}], [8.582526351\ 10^{-1009}, 8.582526351\ 10^{-1008}], [8.582526351\ 10^{-1010}, 8.582526351\ 10^{-1009}], [8.582526351\ 10^{-1011}, 8.582526351\ 10^{-1010}], [8.582526351\ 10^{-1012}, 8.582526351\ 10^{-1011}]]$$

```
> F:=.9*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],1000,1010)
```

$$[[5.489299059\ 10^{-48}, 6.099221177\ 10^{-48}], [4.940369153\ 10^{-48}, 5.489299059\ 10^{-48}], [4.446332238\ 10^{-48}, 4.940369153\ 10^{-48}], [4.001699014\ 10^{-48}, 4.446332238\ 10^{-48}], [3.601529113\ 10^{-48}, 4.001699014\ 10^{-48}], [3.241376202\ 10^{-48}, 3.601529113\ 10^{-48}], [2.917238582\ 10^{-48}, 3.241376202\ 10^{-48}], [2.625514724\ 10^{-48}, 2.917238582\ 10^{-48}], [2.362963252\ 10^{-48}, 2.625514724\ 10^{-48}], [2.126666927\ 10^{-48}, 2.362963252\ 10^{-48}], [1.914000234\ 10^{-48}, 2.126666927\ 10^{-48}]]$$

Next point of interest was the activity at k=1, should have no stability there, nor an orbit.

```
> F:=1.0*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],1000,1010)
```

[[0.0004943899675, 0.0004948799013], [0.0004939010033, 0.0004943899675], [0.0004934130060, 0.0004939010033], [0.0004929259727, 0.0004934130060], [0.0004924399005, 0.0004929259727], [0.0004919547865, 0.0004924399005], [0.0004914706280, 0.0004919547865], [0.0004909874221, 0.0004914706280], [0.0004905051661, 0.0004909874221], [0.0004900238571, 0.0004905051661], [0.0004895434922, 0.0004900238571]]

```
SFP(A,[z[1],z[2]])
```

$\emptyset$

That behavior remained in line with what we saw in the first order case, stability is lost. This is of interest to us, because it seemed even if we increased how many timesteps ahead we iterate this, the numbers seem to be getting closer and closer to 0, but at a very slow rate. Though still the SFP function says it has no stability. I decided to check if it is semi-stable. I checked numerically through Orb, when starting at [.3,.4] even after 1000010 timesteps we appear closer and closer to [0,0], and when I started at [-.3,-.4], the only time throughout this whole project I change constraints on x, we get further away from [0,0].

```
> F:=1.0*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[-.3,-.4],1000000,1000010)
```

[[Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)], [Float(-∞), Float(-∞)]]

That itself indicates semi stability, but I also wanted to check the eigenvalues.

```
> a1:=diff(z[1]*(1-z[1])*(1-z[2]),z[1])
```

$$a1 := (1-z_1)(1-z_2) - z_1(1-z_2)$$

```
> a2:=diff(z[1]*(1-z[1])*(1-z[2]),z[2])
```

$$a2 := -z_1(1-z_1)$$

```
> a21:=diff(z[1],z[1])
```

$$a21 := 1$$

```
> a22:=diff(z[1],z[2])
```

$$a22 := 0$$

```
> a11:=subs({z[1]=0,z[2]=0},a1)
```

$$a11 := 1$$

```
> a12:=subs({z[1]=0,z[2]=0},a2)
```

$$a12 := 0$$

```
> with(LinearAlgebra):
> J:=Matrix([[a11,a12],[a21,a22]])
```

$$J := \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

```
> evalf(Eigenvalues(J))
```

$$\begin{bmatrix} 0. \\ 1. \end{bmatrix}$$

As we know, since not all eigenvalues here have an absolute value less than 1, than we do not have a stable fixed point. Though we do see that one of the eigenvalues is 1, that is an indicator of possible semi stability, which we do see.

To continue, like the first order case after this value you'd expect to see stability regained after k=1. That remained true even for this case, we see stability immediately come back at k=1.1

```
> F:=1.1*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],1000,1010)
```

[[0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082], [0.04653741082, 0.04653741082]]

We maintain the stability for a good amount of time, in fact we don't lose it until we're near a k value of 4. This is very different from the first order original logistic equation. At a k value of 3.9 we still maintain stability even, but then at 4.0 we lose it.

```
> F:=3.9*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> SFP(A,[z[1],z[2]])
```

{[0.4936303165, 0.4936303165]}

```
> F:=4.0*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> SFP(A,[z[1],z[2]])
```

$\emptyset$

So, we must have a bifurcation in between! Again, through rigorous testing, but this time using SFP instead of Orb, I look for the bifurcation value that brings us to a periodic orbit. I find it at k=3.9999999995.

```
> F:=3.9999999994*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> SFP(A,[z[1],z[2]])
```

{[0.5000000000, 0.5000000000]}

```
> F:=3.9999999995*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> SFP(A,[z[1],z[2]])
```

$\emptyset$

The thing is though, prior to this value, we have stability but at this value we do not display period 2 behavior. In fact, the behavior shown is more like that of a period of 4 orbit.

```
> F:=3.9999999995*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],10000,10010)
```

[[0.4507543032, 0.4976881172], [0.4974391788, 0.4507543032], [0.5492312896, 0.4974391788], [0.4976885544, 0.5492312896], [0.4507590768, 0.4976885544], [0.4974396908, 0.4507590768], [0.5492265216, 0.4974396908], [0.4976889912, 0.5492265216], [0.4507638484, 0.4976889912], [0.4974402024, 0.4507638484], [0.5492217560, 0.4974402024]]

We seem to skip a period of 2 entirely, or maybe that is such a precise value of k where we display a period 2 behavior then flip to a period 4 behavior, that I am unable to currently find that current k value. From here, this period of 4 is maintained all the way up until k=4.5. At this value, the orbit shows no pattern, I don't see a period of any value that is obvious from first glance. It seems to just disappear somewhere before 4.5.

```
> F:=4.5*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],10000,10030)
[[0.2052537030, 0.4859531780], [0.3773416172, 0.2052537030], [0.8402829908, 0.3773416172], [0.3760443734, 0.8402829908], [0.1686384039, 0.3760443734], [0.3936521801, 0.1686384039], [0.8929701756,
  0.3936521801], [0.2607810930, 0.8929701756], [0.09284670464, 0.2607810930], [0.2801771781, 0.09284670464], [0.8232875824, 0.2801771781], [0.4712558553, 0.8232875824], [0.1981444501, 0.4712558553],
  [0.3780385919, 0.1981444501], [0.8484147850, 0.3780385919], [0.3599490438, 0.8484147850], [0.1571538167, 0.3599490438], [0.3815050771, 0.1571538167], [0.8949469640, 0.3815050771], [0.2616703767,
  0.8949469640], [0.09133263238, 0.2616703767], [0.2757361543, 0.09133263238], [0.8165973496, 0.2757361543], [0.4881158312, 0.8165973496], [0.2062114200, 0.4881158312], [0.3770524538, 0.2062114200],
  [0.8390167119, 0.3770524538], [0.3786303287, 0.8390167119], [0.1704349893, 0.3786303287], [0.3953409025, 0.1704349893], [0.8923706559, 0.3953409025]]
```

The orbit maintains that same behavior then once we get to 4.9, Maple can no longer calculate the values our equation takes on.

Next, I again tried to see the activity with negative values of k. Activity was the same for -1<k<0, meaning we found stability at [0,0]. We lose it at k=-1. All things we have seen before, no surprises to us. An orbit of period 2 is reached almost instantly, specifically when we have k=-.99999999995.

```
> F:=-.99999999994*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> SFP(A,[z[1],z[2]])
                                        {[0., 0.]}
> F:=-.99999999995*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> SFP(A,[z[1],z[2]])
                                        ∅
```

From there, like we've seen with the negative values of k before, we reach our bifurcation values very quickly. We are in a period 4 orbit by the time we hit -1.3. Through rigorous testing with Orb we find that the period 4 starts at approximately k=-1.297001.

```
> F:=-1.297001*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],1000,1010)
[[0.4785295935, −0.4785295929], [−0.4785295929, 0.4785295935], [0.4785295923, −0.4785295929], [−0.4785295929, 0.4785295923], [0.4785295935, −0.4785295929], [−0.4785295929, 0.4785295935],
  [0.4785295923, −0.4785295929], [−0.4785295929, 0.4785295923], [0.4785295935, −0.4785295929], [−0.4785295929, 0.4785295935], [0.4785295923, −0.4785295929]]
```

Next, came period 8, bifurcation value for that was at around -1.450906.

```
> F:=-1.450906*z[1]*(1-z[1])*(1-z[2]):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.3,.4],1000,1010)
[[0.3270507520, −0.4691359693], [−0.4691359697, 0.3270507520], [0.6729492500, −0.4691359697], [−0.4691359685, 0.6729492500], [0.3270507497, −0.4691359685], [−0.4691359677, 0.3270507497],
  [0.6729492484, −0.4691359677], [−0.4691359693, 0.6729492484], [0.3270507520, −0.4691359693], [−0.4691359697, 0.3270507520], [0.6729492500, −0.4691359697]]
```

From here I decided to now check the behavior of the equation with extra parameters. Call these parameters a, b, and c. The equation takes on this form:

$$x(n) = kx(n-1)^a (1 - x(n-1))^b (1 - x(n-2))^c$$

With this equation and set of parameters, there was not much I could see through my analysis in regard to behavior on the periodic orbits. I'd think I would need more time to fully do an evaluation on the possible periods, but regardless I will share what I managed to find. The parameter which really seemed to make the biggest difference out of the four, was a. The value of a really determined whether I would have stability or not. For values 0<a<1, we reached stable fixed points, it seemed regardless of the values I gave k, b, and c. When 0<k<1 and 0<a<1, we would not reach a stable fixed point of [0,0] like we would in every other example, we would reach stability at a particular point.

```
> F:=.4*(z[1]^.5)*((1-z[1])^.3)*((1-z[2])^.4):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370],
    [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370], [0.1313681370, 0.1313681370]]
```

```
> F:=.4*(z[1]^.5)*((1-z[1])^2)*((1-z[2])^3):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160,
    0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160], [0.07409456160, 0.07409456160]]
```

This was unlike any behavior we had seen prior in the orbits for the equations, as we always saw if 0<k<1 then we would have stability at [0,0], but not in this case. Even if k=1, where in every other case we would lose stability at this point, as long 0<a<1 we maintained a stable fixed point.

```
> F:=1*(z[1]^.5)*((1-z[1])^.3)*((1-z[2])^.4):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459],
    [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459], [0.4419351459, 0.4419351459]]
```

```
> F:=1*(z[1]^.99)*((1-z[1])^1)*((1-z[2])^1):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454,
    0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454], [0.01949535454, 0.01949535454]]
```

What also was interesting with the activity of 'a', was once 'a' reached 1 and above, the orbit makes its way to 0 very quickly.

```
> F:=1*(z[1]^1.1)*((1-z[1])^1)*((1-z[2])^1):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
                    [[0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.]]
> F:=1*(z[1]^1.001)*((1-z[1])^1)*((1-z[2])^1):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[1.357801392 10^-54344, 2.644928008 10^-54290], [6.151331012 10^-54399, 1.357801392 10^-54344], [2.458997397 10^-54453, 6.151331012 10^-54399], [8.672590364 10^-54508, 2.458997397 10^-54453],
    [2.698279752 10^-54562, 8.672590364 10^-54508], [7.404880840 10^-54617, 2.698279752 10^-54562], [1.792204592 10^-54671, 7.404880840 10^-54617], [3.825083864 10^-54726, 1.792204592 10^-54671],
    [7.198195048 10^-54781, 3.825083864 10^-54726], [1.194210417 10^-54835, 7.198195048 10^-54781], [1.746457138 10^-54890, 1.194210417 10^-54835]]
```

This was even true for higher values of k, though as k grew higher, you also may have needed to increase the value of a to see this behavior again. Unfortunately, I was not able to find a definitive relationship on these two parameters, where I would be able to predict values of a based-on k where you'd get an orbit of [0,0]. Though I did manage to find for example that when k=2, the value that 'a' needed to reach a stable fixed point of [0,0] was 1.2097.

```
> F:=2*(z[1]^(1.2097))*((1-z[1])^(1))*((1-z[2])^(1)):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
```

$$[[0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.]]$$

Time to speak a bit on the behaviors of 'b' and 'c'. Both these parameters did not seem to have any way to counteract the activity of a, regardless of how high you bring their values or how low. The orbit seemed to depend a lot more on 'a'.

```
> F:=1*(z[1]^2)*((1-z[1])^10)*((1-z[2])^9):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
```

$$[[0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.]]$$

```
> F:=1*(z[1]^2)*((1-z[1])^(-10))*((1-z[2])^(-9)):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
```

$$[[0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.], [0., 0.]]$$

With that said, it proved a bit difficult to find periods only because 'a' was having such an influence that it was difficult to see what b and c could do to the orbit. There were things to note regardless. When 'a' took on a value -1<a<0, the equation started to display an orbit with complex numbers. This was regardless of the value on k, and the influence that b and c had was determining how quickly we would move through that complex orbit.

```
> F:=1.1*(z[1]^(-.5))*((1-z[1])^(2))*((1-z[2])^(3)):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10,30)
```

$$\begin{aligned}&[[16143.37242 + 1283.031044\,I, -15.46078115 - 7.180160930\,I], [2.837626405\,10^{9} + 1.281771334\,10^{10}\,I, 16143.37242 + 1283.031044\,I], [4.507550756\,10^{27} - 5.389280999\,10^{27}\,I, 2.837626405\,10^{9}\\
&+ 1.281771334\,10^{10}\,I], [1.353277113\,10^{72} - 5.629190379\,10^{71}\,I, 4.507550756\,10^{27} - 5.389280999\,10^{27}\,I], [6.751602582\,10^{191} - 4.901276301\,10^{190}\,I, 1.353277113\,10^{72} - 5.629190379\,10^{71}\,I],\\
&[-5.321896900\,10^{503} + 1.854162357\,10^{504}\,I, 6.751602582\,10^{191} - 4.901276301\,10^{190}\,I], [7.629249287\,10^{1331} - 5.037067927\,10^{1331}\,I, -5.321896900\,10^{503} + 1.854162357\,10^{504}\,I],\\
&[2.537275173\,10^{3509} + 6.897326043\,10^{3510}\,I, 7.629249287\,10^{1331} - 5.037067927\,10^{1331}\,I], [-1.298908584\,10^{9262} - 7.971503283\,10^{9261}\,I, 2.537275173\,10^{3509} + 6.897326043\,10^{3510}\,I],\\
&[-5.136507938\,10^{24425} - 4.462956260\,10^{24425}\,I, -1.298908584\,10^{9262} - 7.971503283\,10^{9261}\,I], [-8.856946211\,10^{64424} - 1.998012939\,10^{64425}\,I, -5.136507938\,10^{24425} - 4.462956260\,10^{24425}\,I],\\
&[2.373884327\,10^{169915} - 2.627369870\,10^{169915}\,I, -8.856946211\,10^{64424} - 1.998012939\,10^{64425}\,I], [-1.436721431\,10^{448149} + 1.946846070\,10^{448149}\,I, 2.373884327\,10^{169915} - 2.627369870\,10^{169915}\,I],\\
&[-4.042476384\,10^{1181970} - 4.176610517\,10^{1181970}\,I, -1.436721431\,10^{448149} + 1.946846070\,10^{448149}\,I], [2.182451809\,10^{3117404} - 6.929073059\,10^{3117402}\,I, -4.042476384\,10^{1181970}\\
&- 4.176610517\,10^{1181970}\,I], [-4.935219964\,10^{8222018} + 4.921936445\,10^{8222018}\,I, 2.182451809\,10^{3117404} - 6.929073059\,10^{3117402}\,I], [2.013544910\,10^{21685241} + 6.217713417\,10^{21685240}\,I,\\
&-4.935219964\,10^{8222018} + 4.921936445\,10^{8222018}\,I], [-1.174801518\,10^{57193918} - 3.406506191\,10^{57193918}\,I, 2.013544910\,10^{21685241} + 6.217713417\,10^{21685240}\,I], [2.644747448\,10^{150846601}\\
&+ 6.526137677\,10^{150846601}\,I, -1.174801518\,10^{57193918} - 3.406506191\,10^{57193918}\,I], [2.143893758\,10^{397851658} - 2.156957967\,10^{397851658}\,I, 2.644747448\,10^{150846601} + 6.526137677\,10^{150846601}\,I],\\
&[1.466685746\,10^{1049317293} - 1.413520085\,10^{1049317293}\,I, 2.143893758\,10^{397851658} - 2.156957967\,10^{397851658}\,I]]\end{aligned}$$

For this, I had to change my values of K1 and K2 in Orb to 10 and 30, respectively. Otherwise, Maple would just spit out values of undefined float, nothing I could really work with. This was something we've never seen prior. Originally, I thought this only happened with certain bounds on a, but through further numerical analysis I saw that even if a is positive, if b or c is negative enough this complex orbit will show up.

```
> F:=1*(z[1]^.5)*((1-z[1])^(-10))*((1-z[2])^.4):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[-0.2082947259 - 0.2227373119 I, -0.01620044006 - 0.1525716901 I], [-0.06954347223 - 0.01532620806 I, -0.2082947259 - 0.2227373119 I], [0.005627389154 - 0.1476739004 I,
  -0.06954347223 - 0.01532620806 I], [-0.2309389894 - 0.2950276412 I, 0.005627389154 - 0.1476739004 I], [-0.05599378182 + 0.01545457837 I, -0.2309389894 - 0.2950276412 I],
  [-0.01620043955 + 0.1525716893 I, -0.05599378182 + 0.01545457837 I], [-0.2082947237 + 0.2227373135 I, -0.01620043955 + 0.1525716893 I], [-0.06954347332 + 0.01532620745 I,
  -0.2082947237 + 0.2227373135 I], [0.005627389231 + 0.1476738999 I, -0.06954347332 + 0.01532620745 I], [-0.2309389875 + 0.2950276420 I, 0.005627389231 + 0.1476738999 I],
  [-0.05599378204 - 0.01545457877 I, -0.2309389875 + 0.2950276420 I]]
```

Though as you'll see it shows up much slower, as this is to a K2 value of 10010. Furthermore, I tested the same bounds on a that gave us a complex orbit and used them for b and c, and I got the same results. It seems if at least one of a, b, or c is between -1 and 0, we will reach complex values in our orbit very quickly for the most part. The closer the value is to -1 the quicker we see the complex values. Another thing to note, it seems that 'c' had the most impact on whether we'd reach stability more than 'b' did. In the image, I have provided you'll notice that with a simple switch of the decimal from c to b, we come from a stable fixed point and then once the decimal moves from c to b, we reach an orbit with no noticeable period.

```
> F:=.646*(z[1]^(.2))*((1-z[1])^(5))*((1-z[2])^(.7)):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476],
  [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476], [0.1630034476, 0.1630034476]]
> F:=.646*(z[1]^(.2))*((1-z[1])^(.5))*((1-z[2])^(7)):
> A:=ToSys(2,z,F)[1]:
> Orb(A,[z[1],z[2]],[.5,.5],10000,10010)
[[0.09588206350, 0.01766431366], [0.3392438511, 0.09588206350], [0.2088964779, 0.3392438511], [0.02310063370, 0.2088964779], [0.05827944899, 0.02310063370], [0.3014709189,
  0.05827944899], [0.2790154068, 0.3014709189], [0.03448359903, 0.2790154068], [0.03278058904, 0.03448359903], [0.2508626228, 0.03278058904], [0.3357959617, 0.2508626228]]
```

I did find other things, but not much too different from what I've already shown. I had a lot of issues trying to see periodic orbits, maybe through further analysis someday I can find them.

Lastly, I worked on numerical analysis of the first order original discrete logistic equation, but with two more parameters a and b. The equation took on the form:

$$x(n) = k\,x(n-1)^{a}\,(1 - x(n-1))^{b}$$

The things I tested to see first were if it followed behavior similar to that of the previous case with the second order equation and parameter c. I checked values of a between 0 and 1, saw that we would still reach a stable fixed point. This stayed true until k took on values near 1, then we reached a periodic orbit of 2, was very noticeable at k=1.4 and a=.2. With k=1.4 and a=.1 we had no noticeable orbit, Maple couldn't compute only gave us Float(infinity)+Float(infinity)I.

```
> F:=[1.4*(x^(.1))*((1-x)^(1))]:
> Orb(F,[x],[.5],10000,10010)
[[Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) +
    Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I]]
> F:=[1.4*(x^(.2))*((1-x)^(1))]:
> Orb(F,[x],[.5],10000,10010)
    [[0.3226870804], [0.7562644937], [0.3226870804], [0.7562644937], [0.3226870804], [0.7562644937], [0.3226870804], [0.7562644937], [0.3226870804], [0.7562644937], [0.3226870804]]
```

The periodic orbit with these values of k, a, and b kept the orbit until we hit 1.7 for a value of k and we seem to lose all signs of an orbit.

```
> F:=[1.7*(x^(.2))*((1-x)^(1))]:
> Orb(F,[x],[.5],10000,10030)
[[0.1163962620], [0.9769993449], [0.03891956537], [0.8535761146], [0.2411622344], [0.9706434266], [0.04960965728], [0.8860621423], [0.1890644202], [0.9880006683], [0.02034967267],
    [0.7642430446], [0.3798041752], [0.8687397759], [0.2169501747], [0.9806416150], [0.03278084226], [0.8300377281], [0.2783690939], [0.9499226639], [0.08426122903], [0.9491745415],
    [0.08550656158], [0.9506691560], [0.08301820672], [0.9476420068], [0.08805636922], [0.9536062103], [0.07812366068], [0.9411917398], [0.09876950428]]
```

Then at 1.8 Maple can no longer compute the values of the orbit.

```
> F:=[1.8*(x^(.2))*((1-x)^(1))]:
> Orb(F,[x],[.5],10000,10010)
[[Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) +
    Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I], [Float(∞) + Float(∞) I]]
```

Though once you change the value of 'a' to something higher but still a decimal we see different activity in the orbit.

```
> F:=[1.8*(x^(.3))*((1-x)^(1))]:
> Orb(F,[x],[.5],10000,10010)
    [[0.1954762299], [0.8874411678], [0.1954762303], [0.8874411680], [0.1954762299], [0.8874411678], [0.1954762303], [0.8874411680], [0.1954762299], [0.8874411678], [0.1954762303]]
```

Other things of interest were other instances where we do have periodic orbits, with our parameters but with manipulation of one we notice eventually that we lose our periodic orbit completely. Which is unusual, and I don't exactly know what to attribute it to. For example, when we had k=3.9, a=1.1, and b=1.1 we have period 4 orbit.

```
> F:=[3.9*(x^(1.1))*((1-x)^(1.1))]:
> Orb(F,[x],[.5],10000,10010)
    [[0.4701924182], [0.8454691725], [0.4157084045], [0.8222897857], [0.4701924182], [0.8454691725], [0.4157084045], [0.8222897857], [0.4701924182], [0.8454691725], [0.4157084045]]
```

Though when we get to higher values of k, we see that we lose that periodic orbit entirely. Look when k=4.3.

```
> F:=[4.3*(x^(1.1))*((1-x)^(1.1))]:
> Orb(F,[x],[.5],10000,10200)
[[0.2164527095], [0.6107163111], [0.8854922296], [0.3468088248], [0.8396764039], [0.4736840810], [0.9329906269], [0.2037429429], [0.5815913142], [0.9084665044], [0.2788343793],
[0.7365257309], [0.7082510078], [0.7589002185], [0.6638777288], [0.8258713180], [0.5093653774], [0.9354806967], [0.1960042124], [0.5632997416], [0.9193560884], [0.2457702613],
[0.6734490391], [0.8127364348], [0.5421404026], [0.9285321835], [0.2175538022], [0.6131852993], [0.8832275990], [0.3533642267], [0.8476937758], [0.4523980937], [0.9265156190],
[0.2237798711], [0.6269818480], [0.8696647810], [0.3920337810], [0.8879564977], [0.3396445556], [0.8305219554], [0.4974844086], [0.9358157979], [0.1949614861], [0.5608032072],
[0.9206298860], [0.2418715213], [0.6654704222], [0.8237360321], [0.5147723620], [0.9349433184], [0.1976757357], [0.5672860118], [0.9172163044], [0.2523054835], [0.6865693775],
[0.7935547497], [0.5878796480], [0.9040910958], [0.2919756868], [0.7592814711], [0.6630892852], [0.8269209067], [0.5066964920], [0.9356572070], [0.1954550065], [0.5619857523],
[0.9200328887], [0.2436995303], [0.6692254122], [0.8186221144], [0.5275978826], [0.9327061127], [0.2046260585], [0.5836520062], [0.9070682390], [0.2830433760], [0.7439588606],
[0.6939257358], [0.7822079853], [0.6137182994], [0.8827322308], [0.3547944912], [0.8493973150], [0.4478230197], [0.9246377879], [0.2295653086], [0.6395503688], [0.8559723353],
[0.4299962499], [0.9156828380], [0.2569778055], [0.6957538185], [0.7793225877], [0.6201419589], [0.8765811771], [0.3724426006], [0.8690655627], [0.3937182315], [0.8894355696],
[0.3353292013], [0.8248105866], [0.5120551628], [0.9352434597], [0.1967422370], [0.5650621869], [0.9184260836], [0.2486128263], [0.6791950224], [0.8045028881], [0.5620865645],
[0.9199814645], [0.2438569284], [0.6695475717], [0.8181781574], [0.5287030123], [0.9324499914], [0.2054208440], [0.5855018894], [0.9057834833], [0.2869029678], [0.7506551908],
[0.6806649091], [0.8023548136], [0.5672133320], [0.9172564832], [0.2521829367], [0.6863262830], [0.7939224680], [0.5870269863], [0.9047032889], [0.2901422304], [0.7561865865],
[0.6694585247], [0.8183009521], [0.5283974762], [0.9325218065], [0.2051980095], [0.5849836857], [0.9061461960], [0.2858140817], [0.7487776126], [0.6844169618], [0.7967944754],
[0.5803345988], [0.9093022124], [0.2763146369], [0.7320108246], [0.7167483251], [0.7443231836], [0.6932129089], [0.7833260163], [0.6112131744], [0.8850404639], [0.3481187362],
[0.8413059151], [0.4693908350], [0.9319846157], [0.2068644893], [0.5888506103], [0.9033867429], [0.2940830108], [0.7628064958], [0.6557485621], [0.8364555680], [0.4821197900],
[0.9345255016], [0.1989748227], [0.5703706611], [0.9154711778], [0.2576219709], [0.6970072014], [0.7773292045], [0.6245446975], [0.8721725221], [0.3849618898], [0.8814956974],
[0.3583589066], [0.8535714213], [0.4365366785], [0.9192708422], [0.2460309555], [0.6739785136], [0.8119886480], [0.5439711771], [0.9278835066], [0.2195580261], [0.6176570254],
[0.8790002647], [0.3655267999], [0.8616565883], [0.4143720173], [0.9056948943], [0.2871688267], [0.7511122257], [0.6797476111], [0.8036973390], [0.5640129095], [0.9189828308],
[0.2469115262], [0.6757632294], [0.8094517374]]
```

There’s no pattern or anything obvious to us. There are even some instances where we have k=4.1, a=1.1, and we change the value of b from 1.2 to 1.3 and we actually go down from a periodic orbit of 4 to a periodic orbit of 2.

```
> F:=[4.1*(x^(1.1))*((1-x)^(1.2))]:
> Orb(F,[x],[.5],10000,10010)
[[0.4405156358], [0.8288708420], [0.4009666218], [0.8112302316], [0.4405156358], [0.8288708420], [0.4009666218], [0.8112302316], [0.4405156358], [0.8288708420], [0.4009666218]]
> F:=[4.1*(x^(1.1))*((1-x)^(1.3))]:
> Orb(F,[x],[.5],10000,10010)
[[0.4337597202], [0.7810143017], [0.4337597202], [0.7810143017], [0.4337597202], [0.7810143017], [0.4337597202], [0.7810143017], [0.4337597202], [0.7810143017], [0.4337597202]]
```

When we bring b to a high enough value, for example 2.6, we lose the periodic orbit entirely and just keep one stable fixed point throughout the orbit.

```
> F:=[4.1*(x^(1.1))*((1-x)^(2.6))]:
> Orb(F,[x],[.5],10000,10010)
[[0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988], [0.3978431988]]
```

Eventually as b kept getting larger, our values in the orbit remain stable but for every value we bring b up, the numbers get closer and closer to 0, ultimately reaching stability at 0.

```
> F:=[4.1*(x^(1.1))*((1-x)^(12))]:
> Orb(F,[x],[.5],10000,10010)
[[0.09317416899], [0.09317416899], [0.09317416899], [0.09317416899], [0.09317416899], [0.09317416899], [0.09317416899], [0.09317416899], [0.09317416899], [0.09317416899],
[0.09317416899]]
> F:=[4.1*(x^(1.1))*((1-x)^(30))]:
> Orb(F,[x],[.5],10000,10010)
[[0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.]]
```

We notice similar behavior to when we keep ‘b’ to a fixed value and just manipulate the value of ‘a’. For example, keeping k=4.1 and just manipulating a, and keeping b=1. We notice that as a gets higher we go from periodic orbits to stability to a single point.

```
> F:=[4.1*(x^(1.3))*((1-x)^(1))]:
> Orb(F,[x],[.5],10000,10030)
[[0.5201792586], [0.8411274665], [0.5201792656], [0.8411274693], [0.5201792586], [0.8411274665], [0.5201792656], [0.8411274693], [0.5201792586], [0.8411274665], [0.5201792656],
  [0.8411274693], [0.5201792586], [0.8411274665], [0.5201792656], [0.8411274693], [0.5201792586], [0.8411274665], [0.5201792656], [0.8411274693], [0.5201792586], [0.8411274665],
  [0.5201792656], [0.8411274693], [0.5201792586], [0.8411274665], [0.5201792656], [0.8411274693], [0.5201792586], [0.8411274665], [0.5201792656]]
```

```
> F:=[4.1*(x^(1.8))*((1-x)^(1))]:
> Orb(F,[x],[.5],10000,10010)
  [[0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069], [0.6598640069]]
```

Eventually though as a, got bigger we see an eventual stable fixed point at 0. Similar to what we saw in the second order equation, with parameters a, b, and c. There wasn't much else to report with these.

I will finish up by discussing some limitations that I had to deal with my numerical analysis. The cases where we added extra parameters, they require a lot more time dedicated to the analysis of them. I believe a lot of the behavior we see could be explained by ratios between the parameters, but due to a limitation of how much time I have, I could not do the work justice. Other issues that arose were that of, the second order case with parameters a, b, and c added. I believe if I was able to use the function SFP to its full capacity I would be able to find out more info on stability, but it seems that there is a Maple error somewhere that does not like when I gave 'a' values less than 1. There were just error messages that kept popping up in those instances, but it wasn't anything on the actual code for SFP, I believe it is an error to an integral part of Maple functions, that just wouldn't work. Also, I myself am very new to Maple and so I believe my inexperience was also a limitation in finding out more on the behavior of the orbits. The findings that I did see were interesting, though due to limitations on time and it being my first time doing numerical analysis I feel there were some things I may have missed, but if I ever decide to keep working on it. Maybe I will be able to do better work, and find out more about the equations we have.