Project 1

A full Maple implementation of "Dynamic complexity in predator-prey models framed in difference equations" by J.R. Beddington et. al. , Nature v. 255 (1975), pp 58-60

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Over the semester we have studied difference equations in numerous forms, from observing the orbits of systems, to understanding the fixed and equilibrium points to determining the conditions for stability. One of the first multi dimensional difference models we studied was the predator-prey relationship. The effect that predators and prey have on each other in determining their populations at unique time points. We found the article “Dynamic complexity in predator-prey models framed in difference equations” to be a foundational text for these difference equations, and while the calculus supported the theoretics of the research, the numerics were loosely supported via calculations performable with the given equipment. For our Project we have taken the difference equations modeled by Beddington and created Maple procedures and code to further support the conjectures made by the researchers.

Our first item is a Maple implementation of the original Predator-Prey model by J. R. Beddington. While Beddignton supported his ideas using calculus and older computational methods, modern computing simplifies the process and allows for greater visualization. In the original models the stability and equilibrium of the product values for the constant r were found using manual calculations. Using maple we are able to manipulate the equations and observe the change in the orbits to determine the conditions for extinction and stability in the density dependent predator prey model. The parameters listed in the Maple implementation are the Host Carrying Capacity (k), Searching Efficiency of Predators (a), Equilibrium Constant (r), and Equilibrium Prey Population Density (q). By applying the same parameters as used by Beddington et. al. we found the stability of the prey self regulation at constant values of the range $0<r<2$, first bifurcating cycle for $2<r<2.522$, a second bifurcating cycle for $2.522<r<2.653$ chaos for $r>2.653$. The variation in the bifurcating cycles as compared to the paper’s values comes from our ability to test long term stability of the equations without repetitive and strenuous calculations. We were able to test values at a finer scale and clearly observe the variations in this equation.

From the code The list of numbers given by the orbit of the Predator-Prey model is the population density of each species. The first list is the self regulation of prey without predators and the second list is the population of prey and predators. A trend we noticed in the various sub groups of regulation was that between 0-2 the predator prey populations varied uniformly with rises in prey and predator directly related. Between 2-2.522 step by step trends were wide and sometimes were not directly related. The same occurred at geater randomness than $2.522<r<2.653$. At values greater than 2.653 the regulation strayed into chaotic unrelated values.

Our second item is a Maple implementation of the Predator-Prey model by Beddingtion using a maple procedure. This allows users to easily and quickly modify parameters to their

Predator Prey models as they deem necessary without having to delete or rewrite the equations wholly. Two charts that are generated show the population density of the prey over time and the population density of the predators over time. Since we are mapping population density, we chose to use a density plot as you can more clearly see where the values are the most high and the most low.

Our final item is applying the concepts of the predator prey model to the current Lanternfly invasion of Rutgers. This was performed using a difference equation with the parameters being: % of lanternflies producing one egg mass(a) =0.6 and % of lanternflies producing two egg masses (b)=0.3. As the lanterflies on campus are without any known predators beyond the ability of humans to self regulate the flies through physical extermination on Rutgers they need to be modelled using a self regulating predator prey system. We chose the equation displayed based on the reproductive rate and survivability of lanterflies based on known data. We used Orbits to determine the population of the flies over time and found that in a theoretical uncontested model the Lanterflies grew to values of 30+digits within 20 years. As this is purely a mathematical representation we added real world limitations in the form of a carrying capacity. At the realistic carrying capacity levels of 2 million, 5 million and 10 million, the lanterflies at their current pace managed to reach that value within 4 years for all values. This behavior is modelled using point plots of the data over time. The values for the uncontrolled population created a graph with extremely skewed axes which we show with the exponential growth from the first 3 steps. The self regulation point plot of 5 million flies grows rapidly for 4 years before decreasing and stabilizing at the carrying capacity in 10 years.