# Dynamic Models Bio Attendance Quiz -Charles

Question 1: For the SIR model developed by sir ronald Ross, what does "sir" stand for?

The acronym $\boxed{S}\boxed{I}\boxed{R}$ stands for

- number of $\boxed{S}$usceptible Individuals
- Number of $\boxed{I}$nfected Individuals that can spread disease
- Number of $\boxed{R}$ecovered individuals or individuals who died and can no longer spread disease

Question 2: Why does the following property:

"If $y_1(t)$ is a solution to a linear homogeneous ODE, then so is $Cy_1(t)$ given any constant $C$"

not hold for the equation

$$y'(t) = y(t)^2 ?$$

Answer: This is because $y'(t) = y(t)^2$ is a nonlinear Differential equation i.e we might have imaginary roots!

Dynamic Models Bb
Attendance Quiz 3

Question 3: Suppose $a_1$ = fifth digit of RUID (which is 0) is set to 1

$a_2$ is first digit of Ruid - set to 1
$a_3$ is second digit of RUID (set to 8)

$a_1 := 1$
$a_2 := 1$
$a_3 := 8$

Solve by hand the HOMOGENEOUS ODE

$$a_1 y''(t) - a_2 y'(t) + a_3 y(t) = 0$$

with initial conditions: $y(0)=0$, $y'(0)=0$

Shown as: $$y''(t) - y'(t) + 8y(t) = 0$$

Step 1: guess $y = e^{rt}$.

Therefore, $\begin{cases} y'(t) = re^{rt} \\ y''(t) = r^2e^{rt} \end{cases}$

Step 2: Substitute guess into original differential equation to get

$$r^2e^{rt} - re^{rt} + 8e^{rt} = 0$$

# Dynamic Models BB Attendance Quiz 3

Question 3 step 3: factor out $e^{rt}$ to get:

$e^{rt}(r^2 - r + 8) = 0$

Which leaves us with characteristic equation

$r^2 - r + 8 = 0$ aka $r(r-1)$ but shifted up 8

which cannot be factored easily, so using the quadratic formula

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(8)}}{2}$$

when simplified is

$$r_s = \frac{1 \pm \sqrt{1-32}}{2} = \frac{1 \pm \sqrt{-31}}{2}$$

Thus, our candidates for our solutions are:

$r = \frac{1 + \sqrt{-31}}{2}$ and $r = \frac{1 - \sqrt{-31}}{2}$

STEP 4: We construct our general solution

$$y(t) = C_1 e^{\left(\frac{1+\sqrt{-31}}{2}\right)t} + C_2 e^{\left(\frac{1-\sqrt{-31}}{2}\right)t}$$

and given $y(0) = 0$, which implies $t=0$,

$$y(t) = C_1 e^0 + C_2 e^0 = C_1 + C_2 \Rightarrow 0 = C_1 + C_2$$

Dynamic Models Bio
Attendance Quiz 3

Problem 3

and now for $y'(0) = 0$

we

$$y'(0) = 0 = \frac{1+\sqrt{-31}}{2} c_1 e^{\frac{1+\sqrt{-31}}{2}(t=0)} + \frac{1-\sqrt{-31}}{2} c_2 e^{\frac{1-\sqrt{-31}}{2}(t=0)}$$

which implies

$$\frac{1+\sqrt{-31}}{2} C_1 + \frac{1-\sqrt{-31}}{2} C_2 = 0$$

We will include both the result obtained from our general solution and the result obtained above to create the following system:

$$\begin{cases} C_1 + C_2 = 0 \\ \frac{1+\sqrt{-31}}{2} C_1 + \frac{1-\sqrt{-31}}{2} C_2 = 0 \end{cases}$$

Which shows that

$$C_1 = C_2 = 0$$

Therefore, our final solution is $y(t) = 0$

*titute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, Chara*
*ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, Constant*
*ossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimens*
*envalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSpl*
*teEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramS*
*spose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, Is*
*, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve*
*nction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVect*
*odular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, Popo*
*atrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, Row*
*ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, Su*
*e, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, Vector*
*Norm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]*

$$a1 := 1$$

$$a2 := 1$$

$$a3 := 8$$

$D(y)(t) + a3 \cdot y(t) = 0, y(0) = 0, \mathrm{D}(y)(0) = 0\})$

$$y(t) = 0$$

Dynamic Models Bio
Attendance quiz 3

Question 4: Why is the property

if $a(n)$ is a solution, then so is $c * a(n)$ <u>not</u> valid for a nonlinear recurrence equation

$$a(n) = a(n-1)^2$$

trial: $a(n) = r^n$, for any $n$.

☆ When letting $a(n) = r^n$,
it follows that

$$a(n) = a(n-1)^2$$

can be rewritten as:

$$r^n = (r^{n-1})^2.$$

Then, let $k = n-1$. Thus

$$r^{k+1} = (r^k)^2 \Leftrightarrow r^{k+1} = r^{2k}$$

which implies that

$k+1 = 2k$, equivalently, $k = 1$,

or $n = 2$.

Dynam models Bio
Attendance Quiz 3

Therefore, for the hypothetical case

$a(n) = r^n$,

the <u>only</u> time

$a(n) = a(n-1)^2$

is guaranteed to hold true is when $n = 2$.

One instance of an invalid result is when $n = 3$. This yields us $r^3 = r^4$, which would be untrue for any values other than $r = 0$ or $r = 1$, and we never specified what $r$ or $n$ should be.

Dynamic Models Bio Attendance

Quiz 3

Problem 4 might be easier than expected

let $a(n) = 3^n$

therefore, test $a(n) = a(n-1)^2$

and subsequently $c\,a(n) = c(a(n-1))^2$

or $c(3)^n = c(3^{2n+2})$

which yields

$$c(3)^n = 9c(3)^{2n}$$

which implies

$$3^n = 9(3)^{2n}$$

which is not true?