

(a) (i) Continuous time \Rightarrow "rate"

(ii) $x'(t) = -2x(t)$

(iii) $z \rightarrow -2z$

(iv) $z = -2z$

$0 = -3z \Rightarrow z = 0 \Rightarrow$ equilibrium pt.

(v) $z' = -2$

 \hookrightarrow negative \Rightarrow stable!

$F(w) = -2w$

(b) (i) Discrete \Rightarrow pop. changes from generation to generation

(ii) $x(n) = \frac{1}{2}x(n-1)$

(iii) $z \rightarrow \frac{1}{2}z$

(iv) $z = \frac{1}{2}z$

$0.5z = 0 \Rightarrow z = 0$ is a fixed point

(v) $z' = \frac{1}{2}$

$|z'| = |\frac{1}{2}| < 1$

(c) (i) Discrete-time

(ii) $x(n) = 2x(n-1)(1-x(n-1))$

(iii) $z \rightarrow 2z(1-z)$

(iv) $z = 2z - 2z^2$

$2z^2 - z = 0$

$z(2z-1) = 0$

$z = 0, \frac{1}{2} \Rightarrow$ fixed pts

(v) $z' = 2 - 4z$

$z = 0: z' = 2 \Rightarrow |z'| > 1 \Rightarrow$ not SFP

$z = \frac{1}{2}: z' = 2 - 2 = 0 \Rightarrow |0| < 1 \Rightarrow$ SFP

(d) (i) Continuous time

(ii) $x'(t) = 2x(t)(1-x(t))$

(iii) $z \rightarrow 2z(1-z)$

(iv) $z = 2z - 2z^2$

$2z^2 - z = 0$

$z(2z-1) = 0$

$z = 0, \frac{1}{2} \Rightarrow$ equilibrium pt.

(v) $z' = 2 - 4z$

$z'(0) = 2 \neq 0 \Rightarrow$ not negative \Rightarrow unstable

$z'(\frac{1}{2}) = 2 - 2 = 0 \Rightarrow$ not negative \Rightarrow unstable

no stable equilibrium pts.

$F(w) = 2w(1-w)$