Max Mekhanikov - Lecture 10

what is/are fixed point(s) of

$$x(n) = \frac{x(n-1)+4}{x(n-1)+1}$$

which is/are stable and not?

$$x(n) = f(x_{n-1}) \qquad f(x) = x$$

$$f(x) = \frac{x+4}{x+1}, \quad \frac{x+4}{x+1} = x$$

$$x+4 = x^2 + x$$

$$4 = x^2$$

$$x = \pm 2$$

$$f'(x) = \frac{(x+1)-(x+4)}{(x+1)^2} = \frac{-3}{(x+1)^2}$$

$$f'(2) = \frac{-3}{(3)^2} = -\frac{1}{3} \to \text{stable}$$

$$f'(-2) = \frac{-3}{(-1)^2} = 3 \to \text{unstable}$$

Attendance question #1 - Who popularized fractals? Benoit Mandelbrot

Attendance question #2 - Find eigenvectors

$$\det\begin{bmatrix} -3/2-\lambda & 3/2 \\ -3 & 11/4-\lambda \end{bmatrix} = 0 \qquad \vec{V} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$(-3/2-\lambda)(11/4-\lambda) + 9/2 = 0$$

$$\lambda = 1/2, \ \lambda = 3/4$$

$\lambda = 1/2$:

$$A - \lambda I = \begin{bmatrix} -2 & 3/2 \\ -3 & 9/4 \end{bmatrix}$$

$$\downarrow \ r_1 \leftrightarrow r_2$$

$$\begin{bmatrix} -3 & 9/4 \\ -2 & 3/2 \end{bmatrix}$$

$$\downarrow \ \begin{matrix} r_2 \to r_2 - 2/3\, r_1 \\ r_1 \to r_1 \times -1/3 \end{matrix}$$

$$\begin{bmatrix} 1 & -3/4 \\ 0 & 0 \end{bmatrix}$$

$$x = 3/4\, y \longrightarrow \vec{V}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$\lambda = 3/4$:

$$A - \lambda I = \begin{bmatrix} -9/4 & 3/2 \\ -3 & 2 \end{bmatrix}$$

$$\downarrow \ r_1 \leftrightarrow r_2$$

$$\begin{bmatrix} -9/4 & 3/2 \\ 0 & 0 \end{bmatrix}$$

$$\downarrow \ \begin{matrix} r_2 \to r_2 - 3/4\, r_1 \\ r_1 \to r_1 \times -1/3 \end{matrix}$$

$$\begin{bmatrix} 1 & -2/3 \\ 0 & 0 \end{bmatrix}$$

$$x = 2/3\, y \longrightarrow \vec{V}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$