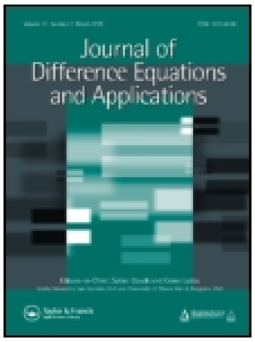

Journal of Difference Equations and Applications

ISSN: 1023-6198 (Print) 1563-5120 (Online) Journal homepage: https://www.tandfonline.com/loi/gdea20

Open problems and conjectures

SI and SIR epidemic models

Gerry Ladas & Linda J.S. Allen

To cite this article: Gerry Ladas & Linda J.S. Allen (2001) Open problems and conjectures, Journal of Difference Equations and Applications, 7:5, 759-761, DOI: 10.1080/10236190108808301

To link to this article: https://doi.org/10.1080/10236190108808301

Published online: 29 Mar 2007.

Journal of Difference Equations and Applications, 2001, Vol. 7, pp. 759–761
Reprints available directly from the publisher
Photocopying permitted by license only

Open Problems and Conjectures

Edited by Gerry Ladas

In this section we present some open problems and conjectures about some interesting types of difference equations. Please submit your problems and conjectures with all relevant information to G. Ladas.

SI and SIR Epidemic Models

LINDA J. S. ALLEN

Texas Tech. University, Lubbock, TX 79409-1042, USA

(Received 30 October 2000; In final form 3 December 2000)

Two conjectures are made about the dynamics of SI and SIR epidemic models.

Keywords: Susceptible; Infected; Epidemic

Cooke *et al.* [3] and Sumpter [4] analyzed an SIR epidemic model of the form:

$$x_{n+1} = x_n(1 - b - c) + y_n(1 - \exp(-ax_n)) \tag{1}$$

$$y_{n+1} = (1 - y_n)b + y_n \exp(-ax_n), \tag{2}$$

where $0 < b+c \leq 1$, $0 < a$, $0 < b$, $0 < c$, $0 < x_0+y_0 \leq 1$, $0 < x_0$ and $0 < y_0$. Infected individuals are x_n and susceptible individuals y_n. Individuals are born susceptible with birth rate b and die at the same rate, infected individuals recover at a rate c and enter the immune or removed group, $1 - x_n - y_n$, and a is the contact rate. For this model, the total population size has been normalized to one. It has been shown that solutions are nonnegative, $0 < x_n + y_n \leq 1$, and if

$$R_0 = \frac{a}{b+c} \leq 1,$$

then solutions approach the infection-free state, that is, $\lim_{n\to\infty}(x_n, y_n) = (0,1)$ [1, 3, 4]. The parameter R_0 is referred to as the basic reproductive number of the disease. In addition, for this model, if $R_0 > 1$, a unique positive endemic equilibrium exists (x^*, y^*), $0 < x^* + y^* < 1$, which is a solution of

$$(b+c)x^* = y^*(1 - \exp(-ax^*)) \quad \text{and} \quad y^* = 1 - x^*(1 + c/b). \qquad (3)$$

The endemic equilibrium appears to be globally asymptotically stable but this has not been verified. For the case $c = 0$ and $x_0 + y_0 = 1$, the SIR epidemic model becomes an SI epidemic model, since there is no recovery. It has been shown for this simpler SI model that if $R_0 > 1$, then

$$\lim_{n\to\infty}(x_n, y_n) = (x^*, y^*)$$

[1, 3, 4].

Another difference equation arising from an SI epidemic model is the second order equation,

$$x_{n+1} = x_n(1-b) + (1-x_n)(1 - \exp(-ax_{n-1})), \quad n = 1, 2, \ldots, \qquad (4)$$

where $0 < b < 1$, $0 < a$ and $0 < x_0$, $x_1 < 1$ [2]. Solutions to this equation satisfy $x_n \in [0, 1]$ and if $R_0 = a/b \leq 1$, it can be shown that $\lim_{n\to\infty} x_n = 0$ [3, 4]. The behavior of this equation in the case $R_0 > 1$ needs further study.

Two conjectures in connection with the SI and SIR epidemic models are stated below.

CONJECTURE 1 *Prove if* $R_0 = a/(b+c) > 1$, *then the solution* (x_n, y_n) *to the first order system* (1) *and* (2) *satisfies*

$$\lim_{n\to\infty}(x_n, y_n) = (x^*, y^*),$$

where x^* *and* y^* *are the positive solutions of* (3).

CONJECTURE 2 *Prove if $R_0 = a/b > 1$, then the solution of (4) satisfies*

$$\lim_{n\to\infty} x_n = x^*,$$

where x^* is the positive solution of

$$bx^* = (1 - x^*)(1 - exp(-ax^*)).$$

References

[1] L. J. S. Allen and A. M. Burgin, Comparison of deterministic and stochastic SIS and SIR models in discrete time. *Math. Biosci.*, **163** (2000), 1–33.
[2] L. J. S. Allen and J. C. Kesinger, Selection in population genetics models of host-pathogen systems. Submitted to the *Proceedings of the Fifth International Conference on Difference Equations and Applications*. Temuco, Chile, January, 2000.
[3] K. L. Cooke, D. F. Calef and E. V. Level, Stability and chaos in discrete epidemic models. In: *Nonlinear Systems and Applications*. V. Lakshmikantham (Ed.) Academic Press, New York, 1977, pp. 73–93.
[4] K. Sumpter, The Dynamics of Some Epidemic Models. *Master's Thesis*. Texas Tech. University, Lubbock, TX, 1995.