> #HW 9 - Alan Ho

> #OK to post

> **read**("M9.txt")

> *Help9*( )

Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

> #1a)

> $f := 2\,x\cdot(1-x)$

$$f := 2\,x\,(1-x) \quad \textbf{(2)}$$

> *Orb*(*f*, *x*, 0.5, 1, 50)

[0.50, 0.5000, 0.50000000, 0.5000000000] **(3)**

> *convert*(%, *set*)

{0.5000000000, 0.50000000, 0.5000, 0.50} **(4)**

> *SFP*(*f*, *x*)

[0.5000000000] **(5)**

> #the stable fixed point is 0.5

> #1b)

> $f := 2.5\,x\cdot(1-x)$

$$f := 2.5\,x\,(1-x) \quad \textbf{(6)}$$

> *Orb*(*f*, *x*, 0.5, 1, 50)

[0.625, 0.5859375, 0.6065368652, 0.5966247410, 0.6016591485, 0.5991635438, 0.6004164790, 0.5997913268, 0.6001042278, 0.5999478590, 0.6000260638, 0.5999869665, 0.6000065162, 0.5999967418, 0.6000016290, 0.5999991855, 0.6000004072, 0.5999997965, 0.6000001018, 0.5999999490, 0.6000000255, 0.5999999872, 0.6000000065, 0.5999999968, 0.6000000015, 0.5999999992, 0.6000000005, 0.5999999998, 0.6000000000] **(7)**

> *convert*(%, *set*)

(8)

{0.5859375, 0.5966247410, 0.5991635438, 0.5997913268, 0.5999478590, 0.5999869665, 0.5999967418, 0.5999991855, 0.5999997965, 0.5999999490, 0.5999999872, 0.5999999968, 0.5999999992, 0.5999999998, 0.6000000000, 0.6000000005, 0.6000000015, 0.6000000065, 0.6000000255, 0.6000001018, 0.6000004072, 0.6000016290, 0.6000065162, 0.6000260638, 0.6001042278, 0.6004164790, 0.6016591485, 0.6065368652, 0.625} **(8)**

> *SFP*(*f*, *x*)

[0.6000000000] **(9)**

> *#the stable fixed point is 0.6*

> *#1c)*

> $f := 3.1\, x \cdot (1 - x)$

$f := 3.1\, x\, (1 - x)$ **(10)**

> *Orb*(*f*, *x*, 0.01, 1, 50)

[0.03069, 0.09221918409, 0.2595158991, 0.5957189313, 0.7465974472, 0.5864880669, 0.7518114243, 0.5784321205, 0.7559300478, 0.5719494129, 0.7589521742, 0.5671256916, 0.7610318386, 0.5637733755, 0.7623921655, 0.5615660896, 0.7632498115, 0.5601685638, 0.7637772061, 0.5593069150, 0.7640963384, 0.5587846844, 0.7642875188, 0.5584715329, 0.7644013474, 0.5582849752, 0.7644688713, 0.5581742699, 0.7645088383, 0.5581087310, 0.7645324637, 0.5580699844, 0.7645464184, 0.5580470968, 0.7645546573, 0.5580335832, 0.7645595199, 0.5580256072, 0.7645623896, 0.5580209002, 0.7645640828, 0.5580181229, 0.7645650819, 0.5580164839, 0.7645656716, 0.5580155167, 0.7645660194, 0.5580149463, 0.7645662246, 0.5580146097, 0.7645663458] **(11)**

> *convert*(%, *set*)

{0.03069, 0.09221918409, 0.2595158991, 0.5580146097, 0.5580149463, 0.5580155167, 0.5580164839, 0.5580181229, 0.5580209002, 0.5580256072, 0.5580335832, 0.5580470968, 0.5580699844, 0.5581087310, 0.5581742699, 0.5582849752, 0.5584715329, 0.5587846844, 0.5593069150, 0.5601685638, 0.5615660896, 0.5637733755, 0.5671256916, 0.5719494129, 0.5784321205, 0.5864880669, 0.5957189313, 0.7465974472, 0.7518114243, 0.7559300478, 0.7589521742, 0.7610318386, 0.7623921655, 0.7632498115, 0.7637772061, 0.7640963384, 0.7642875188, 0.7644013474, 0.7644688713, 0.7645088383, 0.7645324637, 0.7645464184, 0.7645546573, 0.7645595199, 0.7645623896, 0.7645640828, 0.7645650819, 0.7645656716, 0.7645660194, 0.7645662246, 0.7645663458} **(12)**

> *SFP*(*f*, *x*)

[ ] **(13)**

> *#There are no stable fixed points*

> *#1d)*

> $f := \frac{4+x}{3+x}$

$$f := \frac{4+x}{3+x} \quad (14)$$

> $Orb(f, x, 0.01, 1, 50)$

[1.332225914, 1.230828221, 1.236360341, 1.236051686, 1.236068885, 1.236067927, 1.236067980, 1.236067977, 1.236067978] **(15)**

> $convert(\%, set)$

{1.230828221, 1.236051686, 1.236067927, 1.236067977, 1.236067978, 1.236067980, 1.236068885, 1.236360341, 1.332225914} **(16)**

> $SFP(f, x)$

[1.236067977] **(17)**

> *#1.236067977 is the stable fixed point*

> *#1d)*

> $f := \frac{x+4}{x+3}$

$$f := \frac{4+x}{3+x} \quad (18)$$

> $Orb(f, x, 0.01, 1, 50)$

[1.332225914, 1.230828221, 1.236360341, 1.236051686, 1.236068885, 1.236067927, 1.236067980, 1.236067977, 1.236067978] **(19)**

> $convert(\%, set)$

{1.230828221, 1.236051686, 1.236067927, 1.236067977, 1.236067978, 1.236067980, 1.236068885, 1.236360341, 1.332225914} **(20)**

> $SFP(f, x)$

(21)

$$[1.236067977] \quad (21)$$

> #1.236067977 is the stable fixed point

> #1e)

> $f := \frac{3 + x + x^2}{4 + x + 2x^2}$

$$f := \frac{x^2 + x + 3}{2x^2 + x + 4} \quad (22)$$

> $Orb(f, x, 0.01, 1, 50)$

[0.7506109421, 0.7339971848, 0.7352229847, 0.7351331179, 0.7351397095, 0.7351392261, 0.7351392616, 0.7351392590, 0.7351392591] (23)

> $convert(\%, set)$

{0.7339971848, 0.7351331179, 0.7351392261, 0.7351392590, 0.7351392591, 0.7351392616, 0.7351397095, 0.7352229847, 0.7506109421} (24)

> $SFP(f, x)$

$$[0.7351392587] \quad (25)$$

> #0.735139258 is the stable fixed point

>

> #2)

> $f := x \to \frac{x + a}{x + b}$

$$f := x \mapsto \frac{x + a}{x + b} \quad (26)$$

> $solve(f(x) = x, x)$

$$-\frac{b}{2} + \frac{1}{2} + \frac{\sqrt{b^2 + 4a - 2b + 1}}{2}, -\frac{b}{2} + \frac{1}{2} - \frac{\sqrt{b^2 + 4a - 2b + 1}}{2} \quad (27)$$

> $diff\left(-\frac{b}{2} + \frac{1}{2} + \frac{\sqrt{b^2 + 4a - 2b + 1}}{2}, a\right)$

$$\frac{1}{\sqrt{b^2 + 4a - 2b + 1}} \quad (28)$$

> $C(a,b) := \frac{1}{\sqrt{b^2+4a-2b+1}}$

$$C := (a,b) \mapsto \frac{1}{\sqrt{b^2+4\cdot a-2\cdot b+1}} \quad (29)$$

> $Orb(C(1,2), x, 0.9, 1000, 1010)$

$$\left[\frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right] \quad (30)$$

> $FP(C(1,2), x)$

$$[0.4472135954] \quad (31)$$

> $SFP(C(1,2), x)$

$$[0.4472135954] \quad (32)$$

> $Orb(C(2,3), x, 0.9, 1000, 1010)$

$$\left[\frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}, \frac{\sqrt{3}}{6}\right] \quad (33)$$

> $FP(C(2,3), x)$

$$[0.2886751347] \quad (34)$$

> $SFP(C(2,3), x)$

$$[0.2886751347] \quad (35)$$

> $Orb(C(12,17), x, 0.9, 1000, 1010)$

$$\left[\frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}, \frac{\sqrt{19}}{76}\right] \quad (36)$$

> $FP(C(12,17), x)$

$$[0.05735393349] \quad (37)$$

> $SFP(C(12,17), x)$

$$[0.05735393349] \quad (38)$$

>

> *#3)*

#let k=2

> $f := x \to 2x\cdot(1-x)$

$$f := x \mapsto 2\cdot x\cdot(1-x) \quad (39)$$

> $f(x)$

$$2x(1-x) \quad (40)$$

> $solve(f(x)=0, x)$

$$0, 1 \quad (41)$$

> *#0 and 1 are the eqilibrium points*

> $f(-0.999999)$

$$-3.999994000 \quad (42)$$

> $f(-0.25)$

$$-0.6250 \quad (43)$$

> $f(0.0000001)$

$$1.9999998 \times 10^{-7} \quad (44)$$

> $f(0.25)$

$$0.3750 \quad (45)$$

#As seen by plugging in points really close to 0 from both sides, 0 is not a stable equilibrium because the system is moving away from it.

> $SFP(f(x), x)$ *#for k=2*

$$[0.5000000000] \quad (46)$$

> $f := x \to 1\, x \cdot (1 - x)$

$$f := x \mapsto x \cdot (1 - x) \quad (47)$$

> $SFP(f(x), x)$ *#for k=1*

$$[\,] \quad (48)$$

> $f := x \to 3\, x \cdot (1 - x)$

$$f := x \mapsto 3 \cdot x \cdot (1 - x) \quad (49)$$

> $SFP(f(x), x)$

$$[\,] \quad (50)$$

>

> $f := x \to 4\, x \cdot (1 - x)$

$$f := x \mapsto 4 \cdot x \cdot (1 - x) \quad (51)$$

> $SFP(f(x), x)$

$$[\,] \quad (52)$$

> *#k=2 is the bifurcation value, the stable fixed point at k=2 is 0.5*

>

> *#4)*

> $f := x \to k \cdot x \cdot (1 - x)$

$$f := x \mapsto k \cdot x \cdot (1 - x) \quad (53)$$

> $t := f(f(x))$

$$t := k^2 x (1 - x) (1 - k x (1 - x)) \quad (54)$$

> $diff(t, x)$

$$(1 - x) k^2 (1 - k x (1 - x)) - k^2 x (1 - k x (1 - x)) + k^2 x (1 - x) (-(1 - x) k + k x) \quad (55)$$

> $solve(\% = 0, x)$

$$\frac{1}{2}, \frac{k + \sqrt{k^2 - 2k}}{2k}, -\frac{-k + \sqrt{k^2 - 2k}}{2k} \quad (56)$$

> $solve(t = 0, x)$

$$0, 1, \frac{k + \sqrt{k^2 - 4k}}{2k}, -\frac{-k + \sqrt{k^2 - 4k}}{2k} \quad (57)$$

> *#I'm sorry I don't know how to finish this problem, I tried setting* $\frac{k+\sqrt{k^2-4k}}{2k}$ **and**
$-\frac{-k+\sqrt{k^2-4k}}{2k}$ **to** *various things like diff* (t, x)
and 0 *but none of these answers made sense. Will come* **to** *office hours* **for** *this problem.*